# An Adaptive Control Law against Time – Varying Delays in Bilateral Teleoperation Systems

Tran Hoang Anh, Nguyen Anh Tung, Nguyen Thanh Binh, Dao Phuong Nam, Vu Van Tu

Abstract— Bilateral teleoperation is a robotic system helping humans to work with the remote environment through a dual robot which includes a local robot and a remote robot operating with considerable time delays. In order to overcome this obstacle, beside the proposed wave variables and scattering approaches [1], [2], the conventional methods without wave variables has been pointed out in [3], [4] with constant time delay. In this paper, we propose a new adaptive control law based on Lyapunov's direct method to address time varying delays and position synchronization between two robots. In addition, force control was considered to guarantee tracking position error which converges to zero under humans and environment disturbances. The validity of them is based on theory and the good performance of the proposed controller shown in simulation results.

*Keywords*— bilateral teleoperation, adaptive controller, scattering

# I. INTRODUCTION

The Bilateral Teleoperation system includes two robotic manipulators. One is slave robot which is placed in the environment to do tasks. The other one is local robot which is controlled by human to give orders to slave robot. The advantages of this system are that the operator can directly observe and control activities of the slave robot, not only that the slave robot will also send the necessary information to remote help operators giving the timely orders. Besides, this system also has some obstacles. The first one is the position and velocity tracking. This property is very important, because it guaranteed the synchronization between two joints of robots, which make the bilateral teleoperation better than the other robot-systems. Secondly, because of the distance between two robots, there will be communication time-delay between local and remote robot which lead to many other problems.

One of the first model of the bilateral teleoperation was built in 1940s by Goetz. Until 1990s, M. Spong - Anderson, Niemeyer - Slotine [5], [6], [7] gave an idea about using the wave-variable and passive theorem in this system. By using the wave-variable, we can simplify the algorithm and it also gives us an easier way to handle the stability of the system. However, besides the advantages of this method, it still has some disadvantages. Firstly, because of the method's

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characteristic, the dynamic model of the system cannot include the gravity element. So, it needs another independent system to eliminate the gravity element. One another problem of this method cannot deal with the time-varying delays. To overcome these problems, N. Chopra and M. Spong (2004) proposed an adaptive controller for the bilateral teleoperation. Four years later, N. Chopra and M. Spong improved their method by using a new synchronizing signals. Until 2009, E. Nuño and R. Ortega gave an improvement for this method by replaced the adaptive law. Despite of these improvement, the bilateral teleoperation controller still cannot deal with the time-varying delays.

In this paper, we propose a new adaptive control law based on the results of E. Nuño and R. Ortega [4]. Thus, like [4], our controller has all the known properties such as the stability and position tracking. Furthermore, the time-varying problem is also implemented by this adaptive controller to eliminate assumptions of previous researches.

# II. PRELIMINARIES

Throughout the article we use the following notation: .

stand for Euclidean norm and  $\left\| \cdot \right\|_2$  stand for  $\mathcal{L}_2$  norm.

#### A. Dynamic model

The local and remote robots are modeled as a pair of n-Degrees of Freedom (DOF) serial links. Their corresponding nonlinear dynamics, together with the human operator and environment interaction, are described by

$$M_{l}(\mathbf{q}_{1})\ddot{\mathbf{q}}_{1} + C_{l}(\mathbf{q}_{1},\dot{\mathbf{q}}_{l})\dot{\mathbf{q}}_{1} + g_{l}(\mathbf{q}) = \tau_{h} - \tau_{l}$$

$$M_{r}(\mathbf{q}_{r})\ddot{\mathbf{q}}_{r} + C_{r}(\mathbf{q}_{r},\dot{\mathbf{q}}_{r})\dot{\mathbf{q}}_{r} + g_{r}(\mathbf{q}_{r}) = \tau_{r} - \tau_{e}$$
(1)

where  $\ddot{\mathbf{q}}_i, \dot{\mathbf{q}}_i, \mathbf{q}_i \in \mathbb{R}^n$  are the acceleration, velocity and joint position, respectively  $M_i(\mathbf{q}_i) \in \mathbb{R}^{n \times n}$  are the inertia matrices,  $C_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) \in \mathbb{R}^{n \times n}$  are the Coriolis and centrifugal effects,  $g_i(\mathbf{q}_i) \in \mathbb{R}^n$  are the gravitational forces;  $\tau_i \in \mathbb{R}^n$  are the control signals; and  $\tau_h \in \mathbb{R}^n, \tau_e \in \mathbb{R}^n$  are the forces exerted by the human operator and the environment interaction. The subscript *i* stand for *l* or *r*, which are the local or remote robot manipulators, respectively.

These dynamic models have some important well-known properties as following [8] and [9]

P1. The inertia matrix is lower and upper bounded, i.e.,  $0 < \lambda_m \{ M_i \} I \leq M_i (\mathbf{q}_i) \leq \lambda_m \{ M_i \} I < \infty$ 

P2. The Coriolis matrix and inertia are related as  $\dot{M}_i(\mathbf{q}_i) = C_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) + C_i^T(\mathbf{q}_i, \dot{\mathbf{q}}_i)$ 

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P3. The Coriolis forces are bounded as

$$\forall \mathbf{q}_i, \dot{\mathbf{q}}_i \in \mathbb{R}^n \exists k_{c_i} \in \mathbb{R}_{>0}$$

such that  $|C_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i| \leq k_{c_i} |\dot{\mathbf{q}}_i|^2$ 

P4. The dynamics are linearly parameterizable. Thus,  $M_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i + C_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i + g_i(\mathbf{q}_i) = Y_i(\mathbf{q}_i,\dot{\mathbf{q}}_i,\ddot{\mathbf{q}}_i)\theta_i$ where  $Y_i(\mathbf{q}_i, \dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i) \in \mathbb{R}^{n \times p}$  are matrices of known functions and  $\theta_i \in \mathbb{R}^n$  are constant vectors of the manipulator physical parameter.

#### B. Assumptions and definitions

We make the following assumptions

A1. The variable time -- delay, that owing to its nature cannot be negative, has a known upper bound  $T_{M_i}$  i.e.,  $0 < T_i(t) \le T_{M_i}$ . And the derivative of  $T_i(t)$  are also bounded.

A2. We assume that the force of human and environment can be measured and the measured is bound, i.e.,  $|\tau_i - \tilde{\tau}_i| = |\Delta \tau_i| < \varepsilon_i$  where *i* stand for *h* and *e*.

Also, in this paper, we use this definition

D1. For any vector  $\mathbf{v} = \begin{bmatrix} v_1 & v_2 & \dots & v_n \end{bmatrix}^T$  where  $\mathbf{v} \in \mathbb{R}^n$ and  $sign(\mathbf{v}) \in \mathbb{R}^n$  which is defined as

$$sign(\mathbf{v}) = [sign(v_1) \quad sign(v_2) \quad \dots \quad sign(v_n)]^T$$

C. Lemma

We propose this lemma for variable time delays (Nuño et al. 2008, Lemma 1), its proof can be found in [5]

Lemma 1: For any vector signals x, y any variable time delay satisfied A1 and any constant  $\alpha > 0$  we have that

$$-2\int_{0}^{t} \mathbf{x}^{T}(\sigma) \int_{-T(\sigma)}^{0} \mathbf{y}(\sigma+\theta) d\theta d\sigma \leq \alpha \|\mathbf{x}\|_{2}^{2} + \frac{T_{M_{i}}^{2}}{\alpha} \|\mathbf{y}\|_{2}^{2}$$
(2)

## **III. A NEW ADAPTIVE CONTROLLER**

In this paper, we propose a new adaptive controller. Consider the bilateral teleoperation (1), controlled by

$$\begin{aligned} \tau_{r} &= \hat{M}_{r}\left(\mathbf{q}_{r}\right)\lambda\dot{\mathbf{e}}_{r} + \hat{C}_{r}\left(\mathbf{q}_{r},\dot{\mathbf{q}}_{r}\right)\lambda\mathbf{e}_{r} + \hat{g}_{r}\left(\mathbf{q}_{r}\right) - \overline{\tau}_{r} + \tilde{\tau}_{e} \\ \tau_{l} &= -\hat{M}_{l}\left(\mathbf{q}_{l}\right)\lambda\dot{\mathbf{e}}_{l} - \hat{C}_{l}\left(\mathbf{q}_{l},\dot{\mathbf{q}}_{l}\right)\lambda\mathbf{e}_{l} - \hat{g}_{l}\left(\mathbf{q}_{l}\right) + \overline{\tau}_{l} + \tilde{\tau}_{h} \end{aligned}$$
(3)

and for  $Y_i\hat{\theta}_i = -\hat{M}_i(\mathbf{q}_i)\lambda\dot{\mathbf{e}}_i - \hat{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)\lambda\mathbf{e}_i - \hat{g}_i(\mathbf{q}_i)$  then (3) can be written as

$$\tau_{l} = Y_{l} \left( \mathbf{q}_{1}, \dot{\mathbf{q}}_{1}, \mathbf{e}_{1}, \dot{\mathbf{e}}_{1} \right) \hat{\theta}_{l} + \overline{\tau}_{l} + \tilde{\tau}_{h}$$
  
$$\tau_{r} = -Y_{r} \left( \mathbf{q}_{r}, \dot{\mathbf{q}}_{r}, \mathbf{e}_{r}, \dot{\mathbf{e}}_{r} \right) \hat{\theta}_{r} - \overline{\tau}_{r} + \tilde{\tau}_{e}$$

Now we define the synchronizing signals  $\phi_i$  as

$$\phi_i = \dot{\mathbf{q}}_i - \lambda \mathbf{e}_i \tag{4}$$

where  $\mathbf{e}_i$  are defined as:

 $\mathbf{e}_{\mathbf{l}} = \mathbf{q}_{\mathbf{r}} \left( t - T_{r}(t) \right) - \mathbf{q}_{\mathbf{l}}, \mathbf{e}_{\mathbf{r}} = \mathbf{q}_{\mathbf{l}} \left( t - T_{l}(t) \right) - \mathbf{q}_{\mathbf{r}}$  and  $\lambda$  is a positive real scalar.

From (1) and (3), using (4) we have

$$M_{l}(\mathbf{q}_{1})\dot{\phi}_{l}+C_{l}(\mathbf{q}_{1},\dot{\mathbf{q}}_{1})\phi_{l}=Y_{l}\tilde{\theta}_{l}-\overline{\tau}_{l}+\Delta\tau_{h}$$

$$M_{r}(\mathbf{q}_{r})\dot{\phi}_{r}+C_{r}(\mathbf{q}_{r},\dot{\mathbf{q}}_{r})\phi_{r}=Y_{r}\tilde{\theta}_{r}-\overline{\tau}_{r}-\Delta\tau_{e}$$
(5)

The dynamics of the estimations of the uncertain parameters are given by

$$\boldsymbol{\theta}_i = \boldsymbol{\Gamma}_i \boldsymbol{Y}_i^T \boldsymbol{\phi}_i \tag{6}$$

where  $\Gamma_i$  are the positive definite matrices. The torque  $\overline{\tau}_i$ are:

$$\overline{\tau}_{l} = K\phi_{l} + B\dot{\mathbf{q}}_{1} + sign(\phi_{l})\varepsilon_{h}$$

$$\overline{\tau}_{r} = K\phi_{r} + B\dot{\mathbf{q}}_{r} + sign(\phi_{r})\varepsilon_{e}$$
(7)

where K is the positive definite matrix and B is diagonal positive definite matrix.

Theorem 1: Consider the bilateral teleoperator (1) controller by (3) using adaptive law (6) and coordinating torques (7) together with (4). Then, for variable time--delay satisfy A1, all the signals in the system are bounded and position errors converge to zero.

Using the Lemma 1 we will prove the Theorem 1

*Proof:* Let us propose a Lyapunov candidate function V as

$$V = \frac{1}{2} \sum_{i \in \{l,r\}} \left[ \phi_i^T M_i \phi_i + \tilde{\theta}_i^T \Gamma_i^{-1} \tilde{\theta}_i \right] + \frac{1}{2} \lambda B \left| \mathbf{q}_l - \mathbf{q}_r \right|^2$$
(8)

This function is positive definite and radically unbounded in  $\phi_i, \hat{\theta}_i, \mathbf{e}_i$ . Its time derivative along (5) – (7) using P2 is given by:

$$\dot{V} = -\sum_{i \in \{l,r\}} \left[ K \left| \phi_i \right|^2 + B \left| \dot{\mathbf{q}}_i \right|^2 \right] \\ + \lambda B \left( \dot{\mathbf{q}}_1^{\mathrm{T}} \mathbf{q}_1 - \dot{\mathbf{q}}_r^{\mathrm{T}} \mathbf{q}_1 - \dot{\mathbf{q}}_1^{\mathrm{T}} \mathbf{q}_r + \dot{\mathbf{q}}_r^{\mathrm{T}} \mathbf{q}_r \right]$$

from (7) we have

$$\dot{V} = -\sum_{i \in \{l,r\}} \left[ K \left| \phi_i \right|^2 + B \left| \dot{\mathbf{q}}_i \right|^2 \right] + \lambda B \dot{\mathbf{q}}_1^T \left[ \mathbf{q}_r \left( \mathbf{t} - \mathbf{T}_r \left( \mathbf{t} \right) \right) - \mathbf{q}_1 \right] + \lambda B \dot{\mathbf{q}}_r^T \left[ \mathbf{q}_1 \left( \mathbf{t} - \mathbf{T}_1 \left( \mathbf{t} \right) \right) - \mathbf{q}_r \right] + \phi_l^T \Delta \tau_h - \phi_r^T \Delta \tau_e - \phi_l^T sign(\phi_l) \varepsilon_h - \phi_r^T sign(\phi_r) \varepsilon_e e other hand as [10] we have (9)$$

On the other hand, as [10] we have

$$\dot{V} = -\sum_{i \in \{l,r\}} \left[ K \left| \phi_i \right|^2 + B \left| \dot{\mathbf{q}}_i \right|^2 \right] - \lambda B \dot{\mathbf{q}}_{\mathbf{l}}^{\mathsf{T}} \int_{-T_r(t)}^{0} \dot{\mathbf{q}}_r \left( t + \delta \right) d\delta$$
$$- \lambda B \dot{\mathbf{q}}_r^{\mathsf{T}} \int_{-T_l(t)}^{0} \dot{\mathbf{q}}_1 \left( t + \delta \right) d\delta + \phi_l^T \Delta \tau_h - \phi_r^T \Delta \tau_e \qquad (10)$$
$$- \phi_l^T sign(\phi_l) \varepsilon_h - \phi_r^T sign(\phi_r) \varepsilon_e$$



Using Lemma 1 and the fact that

$$\phi_l^T (sign(\phi_l) \varepsilon_h - \Delta \tau_h) \ge 0$$

$$\phi_r^T (sign(\phi_r) \varepsilon_e + \Delta \tau_e) \ge 0$$
(11)

which shown that the integral of (11) is also positive, we consider:

$$V(t) - V(0) \leq -\sum_{i \in \{l,r\}} \left[ K \|\phi_i\|_2^2 + B \|\dot{\mathbf{q}}_i\|_2^2 \right]$$
  
+  $\lambda B \left( \alpha_l + \frac{T_{M_l}}{\alpha_r} \right) \|\dot{\mathbf{q}}_l\|_2^2 + \lambda B \left( \alpha_r + \frac{T_{M_r}}{\alpha_l} \right) \|\dot{\mathbf{q}}_r\|_2^2$   
-  $\int_0^t \phi_l^T \left( sign(\phi_l) \varepsilon_h - \Delta \tau_h \right) d\delta$ 

Take

$$\gamma_{l} = B - \lambda B \left( \alpha_{l} + \frac{T_{M_{l}}}{\alpha_{r}} \right)$$
$$\gamma_{r} = B - \lambda B \left( \alpha_{r} + \frac{T_{M_{r}}}{\alpha_{l}} \right)$$

If there are exists  $\gamma_i > 0$ , then

$$V(0) \ge \sum_{i \in \{r,l\}} \left[ K \left\| \phi_i \right\|_2^2 + \gamma_i \left\| \mathbf{q}_i \right\|_2^2 \right]$$

Thus  $\{\mathbf{q}_1, \mathbf{q}_r\} \in \mathcal{L}_2$ . We can easily see that, with the constants  $\alpha_i > 0$ , we can always choose a proper  $\lambda_i$  satisfy  $\gamma_i > 0$ . This fact together with the Property P1, implies that  $V(t) \leq V(0)$ , thus (8) is bounded. Hence,  $\{\dot{\mathbf{q}}_1, \dot{\mathbf{q}}_r, \mathbf{q}_1 - \mathbf{q}_r\} \in \mathcal{L}_{\infty}$ 

Rewriting  $\mathbf{e}_{l}$  as

$$\mathbf{e}_{\mathbf{l}} = \mathbf{q}_{\mathbf{r}} \left( t - T(t) \right) - \mathbf{q}_{\mathbf{l}} = \mathbf{q}_{\mathbf{r}} \left( t - T(t) \right) - \mathbf{q}_{\mathbf{r}} + \mathbf{q}_{\mathbf{r}} - \mathbf{q}_{\mathbf{l}}$$
Notice that  $\{\mathbf{q}_{\mathbf{l}} - \mathbf{q}_{\mathbf{r}}\} \in \mathcal{L}_{2}$  and  $\mathbf{q}_{\mathbf{r}} - \mathbf{q}_{\mathbf{r}} \left( t - T_{r} \left( t \right) \right) =$ 

$$\int_{0}^{T_{r}(t)} \dot{\mathbf{q}}_{\mathbf{r}} \left( t - \delta \right) d\delta \leqslant T_{r}^{2} \| \dot{\mathbf{q}}_{\mathbf{r}} \|_{2}^{2}$$
 (using Schwartz's inequality). This

Figure 1. System diagram

consider that  $\mathbf{e}_{1} \in \mathcal{L}_{2}$ , and the same manner,  $\mathbf{e}_{r} \in \mathcal{L}_{2}$ . Thus  $\phi_{i}$  and  $\overline{\tau}_{i}$  are bounded. We also have  $\dot{\mathbf{e}}_{i}$  are bounded because  $\dot{\mathbf{q}}_{i}, \dot{T}_{i}(t)$  are all bounded.

Next, we will prove that all the signal in system are converge to zero. Rewrite (1) with the controller (3)

$$\ddot{\mathbf{q}}_{\mathbf{l}} = M_{l}^{-1}(\mathbf{q}_{\mathbf{l}}) \Big[ \hat{M}_{l}(\mathbf{q}_{\mathbf{l}}) \lambda \dot{\mathbf{e}}_{l} + \hat{C}_{l}(\mathbf{q}_{\mathbf{l}}, \dot{\mathbf{q}}_{l}) \lambda \mathbf{e}_{\mathbf{l}} - \overline{\tau}_{l} + \Delta \tau_{h} - C_{l}(\mathbf{q}_{\mathbf{l}}, \dot{\mathbf{q}}_{l}) \dot{\mathbf{q}}_{l} \Big] \ddot{\mathbf{q}}_{\mathbf{r}} = M_{r}^{-1}(\mathbf{q}_{\mathbf{r}}) \Big[ \hat{M}_{r}(\mathbf{q}_{\mathbf{r}}) \lambda \dot{\mathbf{e}}_{\mathbf{r}} + \hat{C}_{r}(\mathbf{q}_{\mathbf{r}}, \dot{\mathbf{q}}_{\mathbf{r}}) \lambda \mathbf{e}_{\mathbf{r}} - \overline{\tau}_{r} - \Delta \tau_{e} - C_{r}(\mathbf{q}_{\mathbf{r}}, \dot{\mathbf{q}}_{\mathbf{r}}) \dot{\mathbf{q}}_{r} \Big]$$
(12)

Because of  $\{\dot{\mathbf{e}}_i, \mathbf{e}_i, \overline{\mathbf{\tau}}_i, \dot{\mathbf{q}}\} \in \mathcal{L}_{\infty}$  together with Properties <u>P1</u> and P3, we conclude that  $\dot{\mathbf{q}}_i \in \mathcal{L}_{\infty}$ . Hence, Barbalat's lemma guarantees that  $\dot{\mathbf{q}}_i \to 0$  as  $t \to \infty$  because  $\ddot{\mathbf{q}}_i \in \mathcal{L}_{\infty}$  $\dot{\mathbf{q}}_i \in \mathcal{L}_2 \cap \mathcal{L}_{\infty}$ 

To point out the position tracking of system, we prove that  $\ddot{\mathbf{q}}_i \rightarrow 0$  when  $\dot{\mathbf{q}}_i \rightarrow 0$ . After differentiating (11) we will have two types of term: the first one contains  $\left(\frac{d}{dt}\right)M_i^{-1}(\mathbf{q}_i)$ times a bounded signal. The other one contains  $M_i^{-1}(\mathbf{q}_i)$ times the derivative of the term in bracket. From the first  $\frac{d}{dt}M_i^{-1} = -M_i^{-1}M_iM_i^{-1} =$ have: term, we  $-M_i^{-1}(C_i + C_i^T)M_i^{-1}$ , which is clearly bounded because of Properties P1 and P3. The derivative of the term in bracket in (10), is bounded because it is a sum of bounded elements. Corollary,  $\left(\frac{d}{dt}\right)\ddot{\mathbf{q}}_i \in \mathcal{L}_{\infty}$  thus  $\ddot{\mathbf{q}}_i$  are uniformly continuous. Using Barbalat's lemma, we conclude that  $\,\ddot{q}_i \rightarrow 0\,.$  Thus  $\lim \left| \mathbf{q}_{\mathbf{l}} - \mathbf{q}_{\mathbf{r}} \left( t - T_{r} \left( t \right) \right) \right| = 0$ This completes the proof.

# IV. SIMULATION

The local and remote manipulators are modeled as a pair of 2-DOF serial links with joints. Their corresponding nonlinear dynamics follow revolute (1). The elements of the inertia matrices  $M_i(\mathbf{q}_i)$  are described as follows:  $M_i$  =  $l_{l_i}^2 m_{l_i} + \left( l_{l_i}^2 + l_{2_i}^2 + 2l_{l_i} l_{2_i} c_{2_i} \right) m_{2_i} \quad , \quad M_{l_{l_i}} = \left( l_{2_i}^2 + l_{l_i} l_{2_i} c_{2_i} \right) m_{2_i} ,$  $M_{i_{11}} = M_{i_{12}}, M_{i_{22}} = l_{2i}^2 m_{2i}$ ; The element of the Coriolis and centrifugal matrices  $C_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)$  are  $C_{i_1} = -l_{1_i}l_{2_i}s_{2_i}\dot{q}_{2_i}m_{2_i}$ ,  $C_{i_{12}} = -l_{1_1}l_{2_1}s_{2_1}(\dot{q}_{2_1} + \dot{q}_{1_1})m_{2_1}, \quad C_{i_{21}} = l_{1_1}l_{2_1}s_{2_1}\dot{q}_{1_1}m_{2_1}, \quad C_{i_{22}} = 0;$ The element of the gravity forces  $g_i(\mathbf{q}_i)$  are  $g_{i_1} = gl_1 c_1 m_1$  $+g(l_{1,}c_{1,}+l_{2,}c_{12,})m_{2_{i}}, g_{i_{1}}=g(l_{1,}c_{1,}+l_{2,}c_{12_{i}})m_{2_{i}}.$  Where  $c_{1}$ ,  $s_1, c_{12}$  are stand for  $\cos(q_1), \sin(q_1), \cos(q_1 + q_1)$ . The parametrization for both local and remote robot are described as follows

$$Y_{i}(q_{i}, \dot{q}_{i}, e_{i}, \dot{e}_{i}) = \begin{bmatrix} \lambda l_{1_{i}}^{2} \dot{e}_{1_{i}} + g l_{1_{i}} c_{1_{i}} & Y_{12_{i}} \\ 0 & Y_{22_{i}} \end{bmatrix}$$

where

 $Y_{12} = \lambda \dot{e}_{2} \left( l_{1}^{2} + 2l_{2}^{2} + 3l_{1} l_{2} c_{2} \right) + \lambda e_{2} \left( -l_{1} l_{2} \dot{q}_{2} s_{2} \right)$  $-l_1 l_2 (\dot{q}_1 + \dot{q}_2) s_2 + g(l_1 c_1 + l_2 c_{12})$  and  $Y_{22} = \lambda \dot{e}_2 (l_1^2 + l_2 c_{12})$  $l_{2}^{2} + l_{1} l_{2} c_{2} + \lambda e_{2} l_{1} l_{2} \dot{q}_{1} s_{2} + g l_{2} c_{12}$ . Then we have the estimated parameters  $\theta_i$  are

$$\theta_i = \begin{bmatrix} m_{1_i} & m_{2_i} \end{bmatrix}^T$$

The lengths for each link of the manipulator are  $l_{\rm L} = 0.3m$ ,  $l_{2_r} = 0.2m, \ l_{1_r} = 0.4m, l_{2_r} = 0.3m$ . The masses for each link correspond to  $m_{l_1} = 4kg, m_{2_1} = 0.5kg, m_{l_1} \cdot 6kg, m_{2_2} = 0.6kg$ . conditions are  $\mathbf{q}_{\mathbf{r}}(0) = \begin{bmatrix} 0.4 & 0.2 \end{bmatrix}^T$ , The initial  $\dot{\mathbf{q}}_{i}(0) = \dot{\mathbf{q}}_{i}(0) = \mathbf{0}; \mathbf{q}_{i}(0) = \begin{bmatrix} 0.8 & 0.7 \end{bmatrix}^{T}.$ 

For this simulation with the controller (3) the parameters used are:  $\lambda = 0.4$ , K = 10 and B = 0.1. The time delay is fluctuated between 0-0.5s.

The human force is generalized as follows



The result of the simulation is described as follows





Figure 3. Parameter estimation of the local and remote manipulators



Figure 4. Joint position of the local and remote manipulators

It can be seen from the figures (4) that the errors between local and remote robots are converge to zero but the over shoot are not small as expected. When humans apply force, the local robot move as the consequent and the remote robot moves follow.

## V. CONCLUSION

In this paper, we have proposed a new adaptive controller for the bilateral teleoperation systems. This controller can be seen as an extension of previous result of E. Nuño, R. Ortega and L. Basañez (2010). This controller assures that, with the bounded varying time-delays, all the signals are bounded, position error and velocity error converge to zero. The simulations performed confirm the conclusions.

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