Robust H-infinity Backstepping Control Design of a Wheeled Inverted Pendulum System

Nguyen Thanh Binh, Nguyen Manh Hung, Nguyen Anh Tung, Dao Phuong Nam and Nguyen Thanh Long

***Abstract*— The issue of applying H_∞ to control wheeled inverted pendulum is a topic of much concern on account of underactuated and nonlinear model. Authors in [1] selected Lyapunov candidate function presented following HJ equation. Almost previous papers using H - infinity to control WIP must assume that desired accelerator is zero and model is linearized at origin, leading to that system does not obtain global asymptotical stability when angular error leave neighborhood of origin. In this paper, we propose a new control method applying H - infinity and Backstepping technique based on Lyapunov direct method to stabilize tracking error to converge to arbitrary ball of origin. The simulation results of WIP under bounded disturbances demonstrate the effectiveness of the proposed controller.**

***Keywords* - H_∞ control, Backstepping design, Wheeled Inverted Pendulum, Linear Matrix Inequalities.**

I. INTRODUCTION

In [2], the Wheeled Inverted Pendulum described includes a pair of identical wheels, a chassis, wheel actuators, an inverted pendulum and a motion control unit, in that the pair of wheels and the inverted pendulum are supported by chassis. The wheel actuators rotate the wheels with respect to the chassis. The wheel actuators are controlled by the motion control unit to move the vehicle and to stabilize the inverted pendulum.

The dynamic model and several control methods are presented in [3]. In this content, we use dynamic model of WIP built by Newton Euler method as [3]. This model is separated into two subsystems, in that the subsystems describe rotation of Cart and straight motion of WIP which is similar to Cart-Pole model. Authors in [4], [5]approached to control inverted pendulum based on energy function, but the disadvantage of them is that disturbances impacting considered system is ignored and pendulum always fluctuate around origin. Olfati-Saber [6] proposed coordinate transformation to change Cart-Pole model to strict forward system to apply nest saturation method. This transformation will not be effective, when some parameters are uncertainties or disturbances appear. Therefore, K.D.Do [2] using the similar transformation in [6] and combines with nest saturation method, disturbance observer to steady error to converge to origin asymptotically with assumption of zero straight accelerator of Cart. Researchers in [7] applied properties of nonholomic system and backstepping technique, but their drawback is assumption of satisfying state constraints. The instantaneous switching of control input is proposed in [8], nevertheless, position of Cart is not able to stabilize at desired point.

In this paper, we apply H_∞ method in [1] for rotation motion and the controller designed via Backstepping technique to control straight motion subsystem. The first controller built by Lyapunov direct method guarantee that tilt angle and angular velocity error converge to neighborhood of origin. In this region, we add the second controller constructed by H_∞ to ensure that all state variables lie in small arbitrary ball of origin. Finally, in order to avoid instantaneous change of control torque phenomenon, we propose virtual control input that is designed by Backstepping technique in [9].

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II. DYNAMIC MODEL

TABLE I. PARAMETER OF WIP

Parameter	Symbol
Distance between two wheels	$D(m)$
Radius of wheel	$R(m)$
Moment of inertia of the wheel about $y-axis$	$J_\omega(kg.m^2)$
Moment of inertia of the chassis and pendulum about $z-axis$	$J_p(kg.m^2)$
Moment of inertia of the chassis about $y-axis$	$J_M(kg.m^2)$
Moment of inertia of heading angle pendulum about $z-axis$	$J_\theta(kg.m^2)$
Mass of pendulum	$m(kg)$
Mass of chassis	$M(kg)$
Mass of wheel	$M_\omega(kg)$
Gravitational acceleration	$g(m/s^2)$
Distance between central point pendulum and chassis	$l(m)$

TABLE II. VARIABLE OF WIP

Heading angle of pendulum	$\theta(rad)$
Tilt angle of pendulum	$\phi(rad)$
Torque control in left and right wheel	$\tau_L, \tau_R\,(N.m)$
Position of chassis	$x(m)$
Disturbances impacting on two wheels	$d_L, d_R\,(N)$

By applying Newton – Euler Approach in [1], the dynamic model of WIP is:

$$\ddot{\theta} = \frac{D}{2RJ_\theta}(\tau_L - \tau_R) + \frac{D}{2J_\theta}(d_L - d_R) \tag{1}$$

$$\begin{aligned} & ml\cos(\phi)\ddot{\phi} + \left[M + m + 2\left(\frac{J_\omega}{R^2} + M_\omega\right)\right]\ddot{x} \\ & = ml\dot{\phi}^2\sin(\phi) + \frac{\tau_L + \tau_R}{R} + d_L + d_R \end{aligned} \tag{2}$$

$$\left(ml^2 + J_M\right)\ddot{\phi} + ml\cos(\phi)\ddot{x} = mgl\sin(\phi) \tag{3}$$

Where

$$J_\theta = J_p + D^2\left(M_\omega + \frac{J_\omega}{R^2}\right) \tag{4}$$

Assumption 1: The External disturbances impacting two wheels are bounded

$$|d_R|_\infty < d_{R\max};\ |d_L|_\infty < d_{L\max} \tag{5}$$

$$\left[M + m + 2\left(\frac{J_\omega}{R^2} + M_\omega\right)\right]\left(ml^2 + J_\omega\right) - m^2l^2 > 0 \tag{6}$$

Control objective: The heading angle, position and their derivative track their desired value and tilt angle converge to zero.

Setting $\theta_d, \dot{\theta}_d, x_d$ and $\dot{x}_d$ are desired heading angle, heading angular velocity, position and velocity respectively. The proposed controller need to satisfy (7) and (8).

$$|\theta - \theta_d| \to 0,\ \phi \to 0, |x - x_d| \to 0 \tag{7}$$

$$\begin{aligned} & |\dot{\theta} - \dot{\theta}_d| \to 0,\ \dot{\phi} \to 0, |\dot{x} - \dot{x}_d| \to 0 \\ & if\ \ d_L = 0,\ d_R = 0 \end{aligned} \tag{8}$$

Moreover, uniformly bounded in tracking error if

$$\begin{aligned} & |d_L|_\infty < d_{L\max}, |d_R|_\infty < d_{R\max} \\ & |\ddot{x}_d|_\infty < x_{d\max}, |\ddot{\theta}_d| < \theta_{\max} \end{aligned} \tag{9}$$

Variable and parameter changes are used as follows:

$$\begin{aligned} & (d_L + d_R) = d;\ (d_L - d_R) = d_\theta \\ & \frac{1}{R}(\tau_L - \tau_R) = \tau_1;\ \frac{1}{R}(\tau_L + \tau_R) = \tau_2 \end{aligned} \tag{10}$$

$$\begin{aligned} & M_{11} = M + m + 2\left(\frac{J_\omega}{R^2} + M_\omega\right) \\ & M_{12} = M_{21} = ml\cos(\phi) \\ & M_{22} = ml^2 + J_M \end{aligned} \tag{11}$$

Substituting (10) and (11) into (1), (2) and (3):

$$\ddot{\theta} = \frac{D}{2J_\theta}\tau_1 + \frac{D}{2J_\theta}d_\theta \tag{12}$$

$$M_{11}\ddot{x} + M_{12}\ddot{\phi} = ml\dot{\phi}^2\sin(\phi) + \tau_2 + d \tag{13}$$

$$M_{21}\ddot{x} + M_{22}\ddot{\phi} = mgl\sin(\phi) \tag{14}$$

III. PROPOSED METHOD

The above equations is separated into two subsystem: θ system (12) and x, ϕ system (13,14)

A. Control design for θ - subsystem

The control input in (12) and the error of heading angle are set as (15)

$$\begin{aligned} & \tau_1 = u_1 + \frac{2J_\theta}{D}\ddot{\theta}_d \\ & \theta_e = \theta - \theta_d \end{aligned} \tag{15}$$

The subsystem (12) is written as follows:

$$\ddot{\theta}_e = \frac{D}{2J_\theta}u_1 + \frac{D}{2J_\theta}d_\theta \tag{16}$$

Considering subsystem (16) as follows:

$$\begin{cases} \dot{\upsilon} = A\upsilon + B_1u_1 + B_2d_\theta \\ y = H\upsilon \end{cases} \tag{17}$$

$$\upsilon = \left[\theta_e, \dot{\theta}_e\right]^T, A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B_1 = B_2 = \begin{bmatrix} 0 \\ \dfrac{D}{2J_\theta} \end{bmatrix}, H > 0$$

Theorem 1: By selecting positive scalar γ, there exists a matrix $P_1 = P_1^T > 0$ satisfying inequality (18) – H_∞ [1]. The control input (19) ensures (20) to guarantee tracking error to converge to origin and to obtain its robust stability.

$$A^TP_1 + P_1A + P_1\left(\frac{1}{\gamma^2}B_2B_2^T - B_1B_1^T\right)P_1 + H^TH \le 0 \tag{18}$$

$$u_1 = -B_1^TP_1\upsilon \tag{19}$$

$$\int_0^T\left(\|y\|^2 + \|u_1\|^2\right)dt \le \gamma^2\int_0^T\|d_\theta\|^2\,dt \tag{20}$$

Proof:

Selecting Lyapunov candidate function:

$$V_1 = \upsilon^TP_1\upsilon \tag{21}$$

$$\frac{dV_1}{dt} = \upsilon^T \left(A^T P_1 + P_1 A \right) \upsilon + 2u_1^T B_1^T P_1 \upsilon + 2 d_\theta^T B_2^T P_1 \upsilon \tag{22}$$

Substituting (19) into (22)

$$\frac{dV_1}{dt} = \upsilon^T \left(A^T P_1 + P_1 A - 2 P_1 B_1 B_1^T P_1 \right) \upsilon + 2 d_\theta^T B_2^T P_1 \upsilon \tag{23}$$

Substituting (18) into (23)

$$\begin{aligned} \frac{dV_1}{dt} \le & -\upsilon^T \left(P_1 \left(\frac{1}{\gamma^2} B_2 B_2^T + B_1 B_1^T \right) P_1 + H^T H \right) \upsilon \\ & + 2 d_\theta^T B_2^T P_1 \upsilon \end{aligned} \tag{24}$$

$$\frac{dV_1}{dt} \le -\frac{1}{\gamma^2} \upsilon^T P_1 B_2 B_2^T P_1 \upsilon - \left\| u_1 \right\|^2 - \left\| y \right\|^2 + 2 d_\theta^T B_2^T P_1 \upsilon \tag{25}$$

$$\frac{dV_1}{dt} \le -\left\| u_1 \right\|^2 - \left\| y \right\|^2 + \gamma^2 \left\| d_\theta \right\|^2 \tag{26}$$

It is clear that (26) derives (20) when (26) integrates both sides from 0 to t. ■

Remark 1: Let $P_1^{-1} = T$, γ is appropriately selected. The existence of solution in (28) satisfies H∞ [1]. The system will be applied input (19) for stabilization.

Multiplying P^{-1} in left and right side of (18)

$$P_1^{-1} A^T + A P_1^{-1} + \left(\frac{1}{\gamma^2} B_2 B_2^T - B_1 B_1^T \right) + \left(H P_1^{-1} \right)^T H P_1^{-1} \le 0 \tag{27}$$

Schur's complement is applying in (27) to be equivalent to (28)

$$\begin{bmatrix} (AT)^T + AT + \frac{1}{\gamma^2} B_2 B_2^T - B_1 B_1^T & (HT)^T \\ HT & -I \end{bmatrix} \le 0 \tag{28}$$

B. Control design for x, ϕ - subsystem

Subsystem (13) and (14) is equivalent to (30) and (31) by using local feedback linearization (29):

$$\begin{aligned} \tau_2 = & -ml\dot{\phi}^2 \sin(\phi) + \frac{m^2 g l^2}{ml^2 + J_M} cos(\phi) \sin(\phi) + \\ & \left(M + m + 2\left(\frac{J_\omega}{R^2} + M_\omega \right) - \frac{m^2 l^2 \cos^2(\phi)}{ml^2 + J_M} \right) \left(u + \ddot{x}_d \right) \end{aligned} \tag{29}$$

$$\ddot{\phi} = -a \cos(\phi) u + a g \sin(\phi) + a \cos(\phi) \left(\Delta - \ddot{x}_d \right) \tag{30}$$

$$\ddot{x}_e = u + \Delta \tag{31}$$

Where

$$a = \frac{ml^2}{ml^2 + J_M}$$

$$\Delta = \frac{d}{M + m + 2\left(\frac{J_\omega}{R^2} + M_\omega \right) - \frac{m^2 l^2 \cos^2(\phi)}{ml^2 + J_M}} \tag{32}$$

The disired position and velocity of WIP are:

$$x_e = x - x_d ; \dot{x}_e = \dot{x} - \dot{x}_d$$

Remark 2: It is difficult to directly design a H∞ controller for subsystems (30) (31) because of underactuated property and interconnect control input in both two equations. We propose the Lyapunov direct method for $\phi, \dot{\phi}$ to lead to given attractor where the linearization model can be applied. The proposed H∞ controller guarantee that $x, \dot{x}$ track to the desired value and $\phi, \dot{\phi}$ are bounded by neighborhood origin simultaneously. Because the proposed method includes two subcontroller, the switching must be employed.

Assumption 2: Disturbances Δ , desired accelerator $\ddot{x}_d$ and parameters satisfy

$$\left| \Delta \right|_\infty < \Delta_{\max} , \left| \ddot{x}_d \right|_\infty < \ddot{x}_{d\max} \tag{33}$$

$$\left[M + m + 2\left(\frac{J_\omega}{R^2} + M_\omega \right) \right] \left(ml^2 + J_\omega \right) - m^2 l^2 > 0 \tag{34}$$

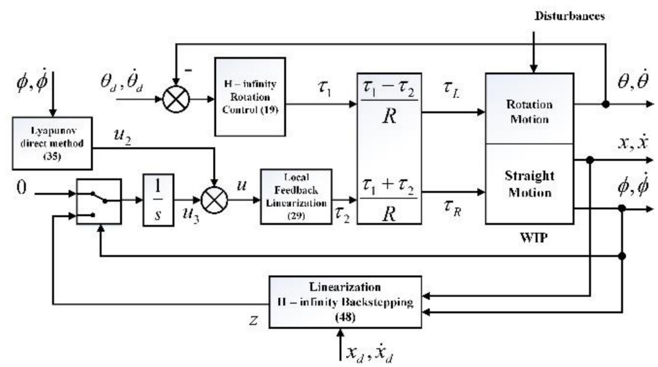

Fig 1. Control Structure for WIP

Theorem 2: Considering subsystem (30) under (33) and (34). By applying control torque (35) found by Lyapunov direct method, two variables $\phi, \dot{\phi}$ converge to the arbitrary attractor $\Omega = \left\{ \Omega \in R^2 \left\| \dot{\phi} \right| \le \varepsilon_1, \left| \phi \right| \le \varepsilon_2 \right\}$ by adjusting controller coefficients k_1, k_2 largely.

$$u_2\left(\dot{\phi}, \phi \right) = \frac{k_1 \dot{\phi} + k_2 \phi + g \sin\phi}{\cos\phi} \tag{35}$$

Proof:

The Lyapunov candidate function is selected:

$$V\left(\phi, \dot{\phi} \right) = \frac{1}{2} \Gamma \phi^2 + \frac{1}{2} \dot{\phi}^2 + \phi \dot{\phi}; \quad \Gamma > 1 \tag{36}$$

The time-derivative of $V\left(\phi, \dot{\phi} \right)$ in (36) is

$$\begin{aligned}\frac{dV}{dt} &= \Gamma\phi\dot{\phi} + \dot{\phi}[-a\cos(\phi)\frac{k_1\dot{\phi}+k_2\phi+g\sin(\phi)}{\cos(\phi)} \\ &+ag\sin(\phi)+a\cos(\phi)(\Delta-\ddot{x}_d)]+\dot{\phi}^2 \\ &+\phi[-a\cos(\phi)\frac{k_1\dot{\phi}+k_2\phi+g\sin(\phi)}{\cos(\phi)} \\ &+ag\sin(\phi)+a\cos(\phi)(\Delta-\ddot{x}_d)]\end{aligned} \tag{37}$$

$$\begin{aligned}\frac{dV}{dt} &= \dot{\phi}\left[-a\left(k_1\dot{\phi}+k_2\phi\right)+a\cos(\phi)(\Delta-\ddot{x}_d)\right] \\ &+\dot{\phi}^2+\Gamma\phi\dot{\phi} \\ &+\phi\left[-a\left(k_1\dot{\phi}+k_2\phi\right)+a\cos(\phi)(\Delta-\ddot{x}_d)\right]\end{aligned} \tag{38}$$

$$\begin{aligned}\frac{dV}{dt} &= -(ak_1-1)\dot{\phi}^2+\phi\dot{\phi}(\Gamma-ak_2-ak_1) \\ &-ak_2\phi^2+(\phi+\dot{\phi})a\cos(\phi)(\Delta-\ddot{x}_d)\end{aligned} \tag{39}$$

Choosing: $\Gamma = ak_2 + ak_1$

$$\begin{aligned}&(39) \Rightarrow \frac{dV}{dt} \le -(ak_1-1)\dot{\phi}^2 - ak_2\phi^2 \\ &+\beta\left(a\cos(\phi)\dot{\phi}\right)^2+\beta\left(a\cos(\phi)\phi\right)^2+\frac{(\Delta-\ddot{x}_d)^2}{4\beta}\end{aligned} \tag{40}$$

Coefficient k_1, k_2 are selected as follows:

$$\begin{aligned}&ak_1 - 1 \ge \beta a^2 + \lambda_1 \ ; \ \lambda_1 > 0 \\ &ak_2 \ge \beta a^2 + \lambda_2 \ ; \quad \lambda_2 > 0\end{aligned} \tag{41}$$

Applying (41) into (40):

$$\frac{dV}{dt} \le -\lambda_1\dot{\phi}^2 - \lambda_2\phi^2 + \frac{\Delta_{\max}^2 + \ddot{x}_{d\max}^2}{2\beta} \tag{42}$$

If we choose λ_1, λ_2, β such that:

$$\varepsilon_1^2 \ge \frac{\Delta_{\max}^2+\ddot{x}_{d\max}^2}{\lambda_1 2\beta},\ \varepsilon_2^2 \ge \frac{\Delta_{\max}^2+\ddot{x}_{d\max}^2}{\lambda_2 2\beta} \tag{43}$$

Then:

$$(\dot{\phi},\phi) \in \Omega := \left\{(x_1,x_2) \in \Omega \subset R^2 \mid |x_1| \le \varepsilon_1, |x_2| \le \varepsilon_2\right\} \tag{44}$$

■

When the angle of pendulum converges to the origin due to control law (35), we linearize (30) and (31) around origin of pendulum where $\phi = 0$ $\dot{\phi} = 0$ and $u = u_2(\phi,\dot{\phi}) + u_3$ as following:

$$\begin{cases}\dot{\eta} = F\eta + Gu_3 + K(\Delta - \ddot{x}_d) \\ \xi = C\eta\end{cases} \tag{45}$$

$$\eta = \begin{bmatrix}\phi \\ \dot{\phi} \\ x_e \\ \dot{x}_e\end{bmatrix} F = \begin{bmatrix}0 & 1 & 0 & 0 \\ -ak_2 & -ak_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ k_2+g & k_1 & 0 & 0\end{bmatrix} G = \begin{bmatrix}0 \\ -a \\ 0 \\ 1\end{bmatrix} K = \begin{bmatrix}0 \\ a \\ 0 \\ 1\end{bmatrix}$$

$$C = diag(\zeta_1,\zeta_2,\zeta_3,\zeta_4) \quad \zeta_i\left(i=\overline{1,4}\right) > 0$$

The Lyapunov function candidate is selected as:

$$V_2 = \eta^T P_2 \eta \tag{46}$$

Applying H∞ - Vandershaft [1] into subsystem (45) and the Lyapunov candidate function (46)

$$F^T P_2 + P_2 F + P_2\left(\frac{1}{\gamma_1^2}KK^T - GG^T\right)P_2 + C^T C \le 0 \tag{47}$$

The control input in (45) is:

$$u_3 = \begin{cases}-G^T P_2 \eta & when\ (\phi,\dot{\phi}) \in \Omega \\ 0 & otherwise\end{cases} \tag{48}$$

Remark 3: Following *Remark 2* and controller (48), the applied switching may cause instantaneous change of control input, leading to unreality. Therefore, a virtual input in (49) is proposed to reduce this problem significantly. The new linear state space model with virtual input is considered as:

$$\begin{cases}\dot{\eta} = F\eta + Gu_3 + K(\Delta - \ddot{x}_d) \\ \dot{u}_3 = z\end{cases} \tag{49}$$

*Theorem 3***:** The proposed virtual control input (49) will be designed and based on backstepping following (50) to satisfy *Remark 3.*

$$z = -\kappa(u_3 - \overline{u}) - \frac{\partial V_2}{\partial \eta}G + \frac{\partial \overline{u}}{\partial \eta}(F\eta + Gu_3) \tag{50}$$

Where $\kappa > 0$ is given, V_2 is the above Lyapunov candidate function designed by H∞ in [1] and $\overline{u}$ is control law generated from (48)

Proof:

Backsteping technique is applied in (49), the third Lyapunov function together (46)

$$V_3 = V_2 + \frac{1}{2}(u_3 - \overline{u})^2 \tag{51}$$

The time-derivative of V_3 is

$$\frac{dV_3}{dt} = \frac{\partial V_2}{\partial \eta}\dot{\eta} + (u_3 - \overline{u})\left(z - \frac{\partial \overline{u}}{\partial \eta}\dot{\eta}\right) \tag{52}$$

$$\begin{aligned}\frac{dV_3}{dt} &= \frac{\partial V_2}{\partial \eta}\left(F\eta + G(u_3 + \overline{u} - \overline{u}) + K(\Delta - \ddot{x}_d)\right) \\ &+(u_3-\overline{u})\left(z - \frac{\partial \overline{u}}{\partial \eta}\left(F\eta + Gu_3 + K(\Delta - \ddot{x}_d)\right)\right)\end{aligned} \tag{53}$$

$$\frac{dV_3}{dt} = \frac{\partial V_2}{\partial \eta}\left(F\eta + G\bar{u} + K\left(\Delta - \ddot{x}_d\right)\right) + \left(u_3 - \bar{u}\right)\left[z + \frac{\partial V_2}{\partial \eta}G - \frac{\partial \bar{u}}{\partial \eta}\left(F\eta + Gu_3 + K\left(\Delta - \ddot{x}_d\right)\right)\right] \quad (54)$$

The virtual input z will be selected as (50), then (54) becomes

$$\frac{dV_3}{dt} = \frac{\partial V_2}{\partial x}\left(F\eta + G\bar{u} + K\left(\Delta - \ddot{x}_d\right)\right) - \kappa\left(u_3 - \bar{u}\right)^2 + \left(u_3 - \bar{u}\right)G^T P_2 K\left(\Delta - \ddot{x}_d\right) \quad (55)$$

$$\frac{dV_3}{dt} \le -\|\xi\|^2 - \|u_3\|^2 + \left(\gamma_1^2 + \frac{1}{4\lambda_3}\right)\left\|\left(\Delta - \ddot{x}_d\right)\right\|^2 - \kappa\|u_3 - \bar{u}\|^2 + \lambda_3\left\|\left(u_3 - \bar{u}\right)G^T P_2 K\right\|^2 \quad (56)$$

$$\frac{dV_3}{dt} \le -\|\xi\|^2 - \|u_3\|^2 + \left(\gamma_1^2 + \frac{1}{4\lambda_3}\right)\left\|\left(\Delta - \ddot{x}_d\right)\right\|^2 - \|u_3 - \bar{u}\|^2\left(\kappa - \lambda_3\left\|G^T P_2 K\right\|^2\right) \quad (57)$$

By choosing $\kappa > \lambda_3\left\|G^T P_2 K\right\|^2$ then we have same results as *Theorem 1*. ■

IV. Simulation

Scenario simulation: The heading angle and tilt angle deviated from their balanced position and position of chassis is arbitrary. The objective of work is to guarantee that $x, \dot{x}, \theta, \dot{\theta}$ track given desired strategies $x_d, \dot{x}_d, \theta_d, \dot{\theta}_d$ with unbalanced initial state, in that WIP system can move to given place via state trajectory. The good performance of simulation results demonstrates the ability of the proposed control law which executes requirement. The parameter values and initial state are used in [2] to simulate this system.

TABLE III. PARAMETER VALUES IN SIMULATION

Parameter	Value
Distance between two wheels	$D = 0.15m$
Radius of wheel	$R = 0.25m$
Moment of inertia of the wheel about $y-axis$	$J_\omega = 1.5kg.m^2$
Moment of inertia of the chassis about $y-axis$	$J_M = 2.5kg.m^2$
Moment of inertia of heading angle pendulum about $z-axis$	$J_\theta = 1.5kg.m^2$
Mass of pendulum	$m = 1.5kg$
Mass of chassis	$M = 5kg$
Mass of wheel	$M_\omega = 1kg$
Gravitational acceleration	$g = 9.8m/s^2$
Distance between central point pendulum and chassis	$l = 1.2m$

TABLE IV. INITIA STATE IN SIMULATION

Initia state variable	Value
θ 0	$0.5rad$
$\dot{\theta}(0)$	$0rad/s$
$x(0)$	$-1.5m$
$\dot{x}(0)$	$0m/s$
$\phi(0)$	$0.8rad$
$\dot{\phi}(0)$	$0.2\,\mathrm{rad/s}$

TABLE V. PARAMETER OF CONTROLLERS

Parameter	value
γ	1.5
H	$diag([100,1])$
u_1	$-[884.43\ \ 477.63][\theta_e, \dot{\theta}_e]^T$
β	100
ε_1	0.3
ε_2	1
λ_1	2.78
λ_2	0.25
k_1	48.41
k_2	39.27
C	$diag([0.1,0.1,1.2,0.1])$
γ_1	100
u_3	$[98.49,31.99,3.44,10.65]\eta$

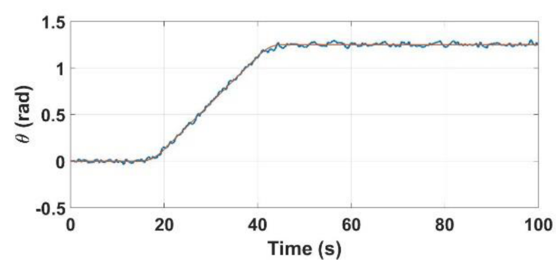

Fig 2. Heading angle

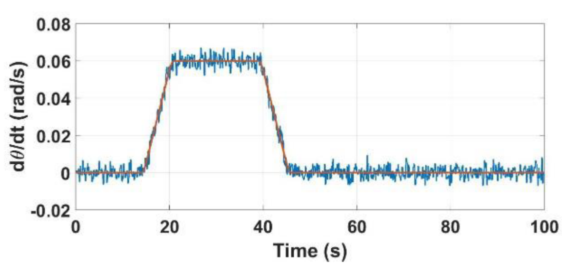

Fig 3. Heading angular velocity

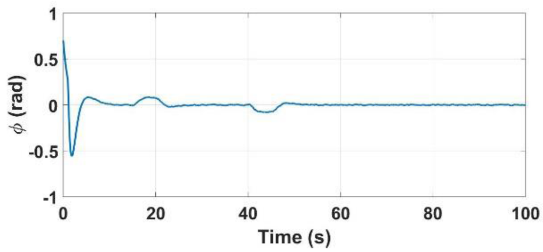

Fig 4. Tilt angle

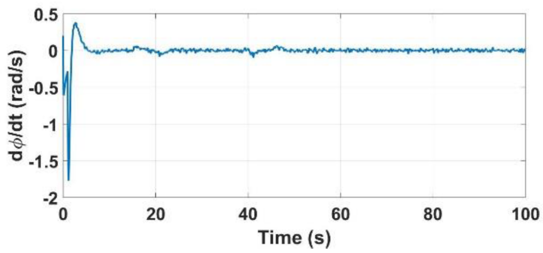

Fig 5. Tilt angular velocity

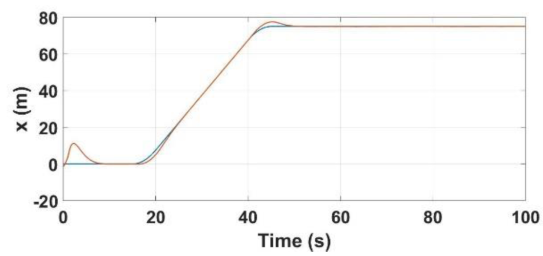

Fig 6. Position of chassis

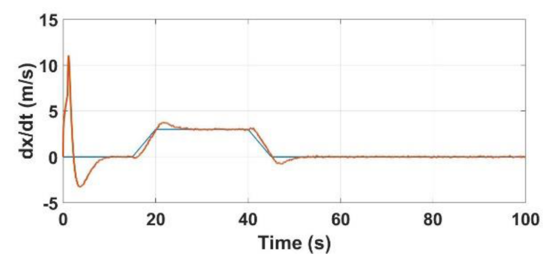

Fig 7. Velocity of chassis

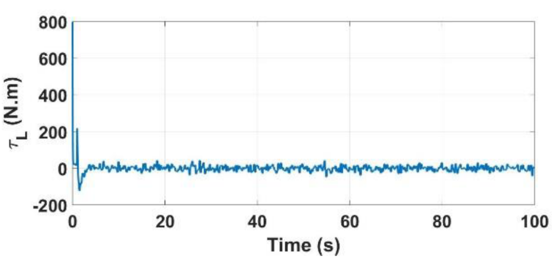

Fig 8. Left torque

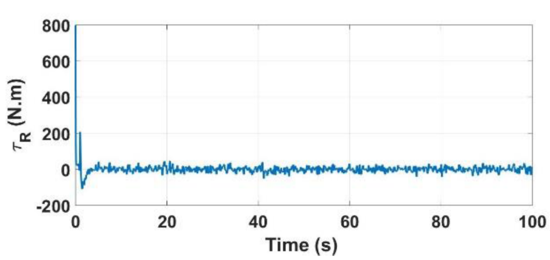

Fig 9. Right torque

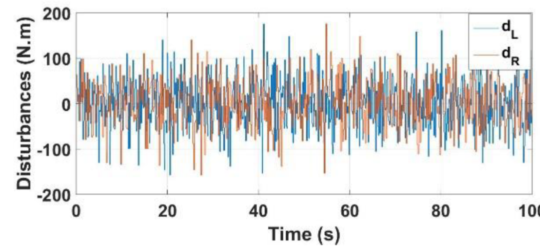

Fig 10. External disturbances impact wheels

This system is able to self-balance and straight velocity, heading angle track reference signal under disturbances. Control forces is continuous.

V. CONCLUSION

The proposed approach applied in this paper is that separating dynamic model into two subsystem including rotation and straight to design two controller. Heading angle tracks its reference when controller is designed by H_∞. Tilt angle and position reach their balanced point by combining two controllers Lyapunov direct method and H_∞. In order to avoid instantaneous change of controllers, the virtual input is applied. The simulation results in previous chapter verify performance of the proposed method.

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