International Journal of Control, Automation and Systems 17(2) (2019) 465-473
http://dx.doi.org/10.1007/s12555-017-0711-0

ISSN:1598-6446 eISSN:2005-4092
http://www.springer.com/12555

An Adaptive Backstepping Trajectory Tracking Control of a Tractor Trailer Wheeled Mobile Robot

Nguyen Thanh Binh, Nguyen Anh Tung, Dao Phuong Nam*, and Nguyen Hong Quang

Abstract: The considered Tractor Trailer Wheeled Mobile Robot (TTWMR) is type of Mobile Robot including a master robot – Tractor and slave robots – Trailers which moves along Tractor to track a given desired trajectory. The main difficulties of the stabilization and the tracking control of TTWMR are due to nonlinear and underactuated systems subjected to nonholonomic constraints. In order to overcome these problems, firstly, we develop the model of TTWMR and transform the tracking error model to the triangular form to propose a control law and an adaptive law. Secondly, the varying time state feedback controllers are designed to generate actuator torques by using Backstepping technique and Lyapunov direct's method, in that these are able to guarantee the stability of the whole system including kinematics and dynamics. In addition, the Babarlat's lemma is used to prove that the proposed tracking errors converge to the origin and the proposed adaptive law is carried on to tackle unknown parameter problem. The simulations are implemented to demonstrate the effective performances of the proposed adaptive law and the proposed control law.

Keywords: Adaptive control, backstepping design, tracking control, tractor trailer wheeled mobile robot.

1. INTRODUCTION

The Tractor Trailer Wheeled Mobile Robot has been playing a crucial role in various applications of the industry such as transportation and delivery systems because of its simplicity, efficiency and flexibility. The considered TTWMR consists of the Tractor which is a unicycle type robot is controlled by two different wheel torques to tow the Trailers tracking a feasible trajectory. By connecting or releasing several Trailers, the system can move payloads following a given path without changing any actuators. It is clear that the TTWMR includes many Wheel Mobile Robots, therefore, it is still the nonlinear and underactuated system subjected to nonholomic constraints, leading to significant complexity in control tasks. In the objective of this paper, the slave robots must track a desired position and orientation with a specified timing law. There were many controllers which were proposed to control single WMR [1–4] , but none of them can apply directly or indirectly to TTWMR because of the difference in kinematic and dynamic model of two types.

Over the last of few decades, several studies have been proposed to prove the stabilization and the tracking control of nonholomic systems such as WMR and TTWMR. In [5], the authors proposed a local feedback transformation to convert many kinematic models of nonholomic system to chained form and the researchers in [6] designed control law to achieve semiglobal tracking by using recursive technique. The main drawback of this approach is that the local diffeomorphism coordinate change in [5] is complicated to apply to other work, in case the movement of the considered TTWMR system is restricted. A kinematic controller in [6] was designed by using exact linearization for the tracking trajectory based on circle path and straight line of articulated vehicle path. All of the above control laws based on mostly kinematic model, assuming perfect velocity tracking and neglecting systems' inertia. There were few reports taking into account of dynamic control approach. In [7], the authors developed the robust adaptive control for the Tractor Trailer system with experiment implementation. The kinematics and dynamics were considered independently to design controllers, hence the stability of the whole system has not been investigated. Kayacan et al. (2015) proposed the nonlinear model predictive control law to obtain trajectory tracking of the Tractor Trailer system [8, 9]. However, it is hard to implement the control law based on microcontroller because of computing ability. The ideal of Backstepping kinematics into dynamics approach was proposed in [10]. However, the authors did not give the specified control law

Manuscript received November 13, 2017; revised July 9, 2018; accepted August 28, 2018. Recommended by Associate Editor Hongyi Li under the direction of Editor Fuchun Sun. This work was supported by Research Foundation funded by Thainguyen University of Technology.

Nguyen Thanh Binh is with the school of Electrical Engineering, University of Ulsan, Ulsan, Korea (e-mail: thanhbinh91@mail.ulsan.ac.kr). Nguyen Anh Tung and Dao Phuong Nam are with Department of Automatic Control, Hanoi University of Science and Technology, Hanoi, Vietnam (e-mails: tung.na134410@sis.hust.edu.vn, nam.daophuong@hust.edu.vn). Nguyen Hong Quang is with the Departement of Automation, Thai Nguyen University of Technology, Vietnam (e-mail: quang.nguyenhong@tnut.edu.vn).
* Corresponding author.

for TTWMR and did not consider unknown parameter problem.

One of the main obstacles of control design in TTWMR is the appearance of unknown parameters in mathematical model, therefore, there were several proposed studies to overcome this challenge. Because of many joint variables in the study [11], the authors divided the general dynamic model into two parts including attitude and position to implement the control law. However, This separation method is not based on the non-holonomic constraints and does not inherit the properties of the electromechanical system, leading to using the Artificial Neural Network (ANN) to estimate the uncertainties. The authors in [12] researched into a fully actuated constrained robot subjected to holonomic constraints. Therefore, they did not need to separate the general dynamic model into two parts to design the control law and the adaptive law based on the ANN as the above study. Furthermore, the theoretical contribution of this research is that finding a solution of the output and full state constraints. The authors in [12] continuously improve their work in [13] when the control law was designed by using the Barrier Lyapunov function. Additionally, the study in [14] consider the fully actuated robotic manipulators subjected to holonomic constrains to design the output feedback control scheme based on the ANN to solve the input saturation problem.

The authors in [15] used the coordinate transforms and Pale approximation to compensate input delays and Barrier Lyapunov function to guarantee the input constraints. In order to implement robust adaptive control law, they used the Backtepping technique based on the ANN to estimate unknown terms. The research [16] did not employ the ANN because the authors investigated the nonlinear strict-feedback systems which is easier than above system. The authors in [15, 16] have improved their work in [17] to research into the SISO stochastic nonlinear system. Because this system contains a simple term which equals the product of unknown constant vector and known nonlinear function vector, the control scheme did not need to use the ANN as [16]. The study in [18] considered using the Fuzzy model to estimate the uncertain terms in all the equations of the state space model. It can be seen that the key point of these above studies [15–18] is to show using Barrier Lyapunov function to design robust adaptive controller.

In this paper, we design a controller which is able to guarantee both the stabilization and the tracking for Tractor – Trailer robot with unknown dynamic parameters. First of all, we develop the model of TTWMR and transform the tracking error model to the triangular form in [7] to propose a control law and an adaptive law. The Backstepping technique [19] and Lyapunov direct method are used to find the control torques and the adaptive law. Finally, that tracking errors converge to the origin is proved by using Babarlat's lemma in [20]. The theoretical contribution in this study focuses on the asymptotic stability of the whole system, while the previous research in [7] only indicates the stability of the control law of each subsystem and ignores the stability of the whole system. In order to overcome the unknown parameter problem, the authors in several studies [11–15, 18] use the Fuzzy control and the ANN to design the adaptive law. Therefore, they are necessary to analyze the Neural Network approximation error in control design, which does not lead to the asymptotic stability, for example, UUB and ISS. On the contrary, because of the separation properties of the electromechanical system, the dynamic model definitely equal to the product of the state functions and the parameters to use the certainty equivalence adaptive control, leading to the asymptotic stability without identifying unknown parameters and using the ANN or the Fuzzy control. Although the estimated parameters of the certainty equivalence cannot converge to the real parameters like the ANN, the tracking errors are able to converge to the origin asymptotically, whose theoretical contribution to the stability proof is absolutely better than that of the ANN.

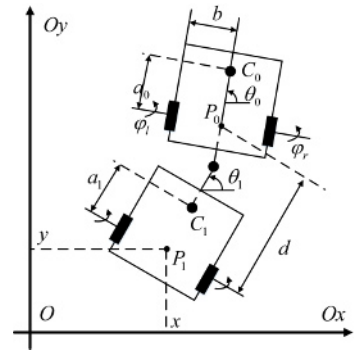

Fig. 1. The model of the investigated TTWMR system.

This paper consists of five parts. In Section 2, problem statement including dynamic model and control objective is introduced. State feedback control design is proposed to prove the stability of the whole system in Section 3. In Section 4, simulation results are implemented to demonstrate the effectiveness of the proposed method in previous section. Final section concludes this paper.

2. PROBLEM STATEMENT

In this section, we introduce the mathematic model of TTWMR and the control objective. Describing the model of TTMWR and non holomic constraints, we use the transformation to eliminate this constraint to build the kinematic and dynamic model. We design a desired trajectory and present several assumption to find two wheel torques to achieve a tracking objective.

Table 1. The parameters and variables of the investigated TTWMR system.

Definition	Symbol (unit)
Distance between master central point and 2 master wheels	a_0 (m)
Distance between slave central point and 2 slave wheels	a_1 (m)
Master and slave mass	m_0, m_1 (kg)
Master and slave inertia	I_0, I_1 (kg.m^2)
Master central point	C_0
Slave central point	C_1
Central point two master wheels	P_0
Central point two slave wheels	P_1
Distance between P_1 and linking joint	d (m)
Distance between $P_0(P_1)$ and wheels	b (m)
Position of Trailer	x (m), y (m)
Reference position of Trailer	x_r (m), y_r (m)
Angle of Tractor	θ_0 (rad)
Reference angle of Tractor	θ_{0r} (rad)
Angle of Trailer	θ_1 (rad)
Reference angle of Trailer	θ_{1r} (rad)
Left and right wheel control torque	τ_l, τ_r (N.m)
Position tracking error	e_x, e_y (m)
Angular tracking error	e_{θ_1}, e_{θ_0} (rad)

2.1. Model description

We consider the TTWMR in Fig. 1 and describe its kinematic model in [8].

$$\dot{q} = S(q)u,$$
$$S(q) = \begin{bmatrix} \cos(\theta_1) & \sin(\theta_1) & \frac{1}{d}\tan(\theta_0-\theta_1) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^T, \quad (1)$$

where $q = [x, y, \theta_1, \theta_0]$ is state variable, u_1 is linear velocity of the Trailer and u_2 is angular velocity of the Tractor. In this paper, $u = [u_1, u_2]$ is associated with angular velocity of two different wheels of Tractor when assuming that robot does not slip and sliding motion between tire and road as follows:

$$u_1 = \frac{r}{2}(\dot{\varphi}_r + \dot{\varphi}_l)\cos(\theta_0 - \theta_1),$$
$$u_2 = \frac{r}{2b}(\dot{\varphi}_r - \dot{\varphi}_l), \quad (2)$$

where r is the radius of two wheels, b is a half of distance between Tractor's wheels, and $\dot{\varphi}_r$, $\dot{\varphi}_l$ are denoted as angular velocities of Tractor's left and right wheels.

The nonholomic system constraints can be written as:

$$A(q)\dot{q} = 0, \quad (3)$$
$$A(q) = \begin{bmatrix} \sin(\theta_0) & -\cos(\theta_0) & -d\cos(\theta_0-\theta_1) & 0 \\ \sin(\theta_1) & -\cos(\theta_1) & 0 & 0 \end{bmatrix}.$$

The dynamic equations of the Tractor – Trailer can be obtained by using the Lagrange method in matrix form as:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} = B(q)\tau + A^T(q)\lambda, \quad (4)$$

where $\tau = [\tau_l, \tau_r]^T$ is the control torque applying to wheels of the Tractor and λ is Lagrange multipliers. Matrices $M(q), C(q,\dot{q}), B(q)$ are same as those in [7] .
From (1), we definitely derive the second time – derivative of the state variable q:

$$\ddot{q} = S(q)\dot{u} + \dot{S}(q)u \quad (5)$$

In order to eliminate the nonholomic constraints in (4), multiplying both side of (4) by matrix $S^T(q)$ and using the fact that $S^T(q)A^T(q) = \Theta$:

$$M_1(q)\dot{u} + C_1(q,\dot{q})u = B_1(q)\tau, \quad (6)$$

where

$$M_1(q) = S(q)^T M(q)S(q), B_1(q) = S(q)^T B(q),$$
$$C_1(q,\dot{q}) = S(q)^T M(q)\dot{S}(q) + S(q)^T C(q,\dot{q})S(q).$$

Setting new control torques:

$$\tau_1 = \frac{\tau_l + \tau_r}{\cos(\theta_0 - \theta_1)},$$
$$\tau_2 = \tau_l - \tau_r. \quad (7)$$

The dynamic model in (6) is written as follows:

$$M_2(q)\dot{u} + C_2(q,\dot{q})u = \bar{\tau}, \quad (8)$$

where

$$\bar{\tau} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} \frac{1}{r} & 0 \\ 0 & \frac{b}{r} \end{bmatrix},$$
$$B_1^{-1}M_1(q) = M_2(q); B_1^{-1}C_1(q,\dot{q}) = C_2(q,\dot{q}),$$
$$M_2(q) = \begin{bmatrix} (m_0+m_1)r + \frac{rI_{\theta_1}}{d^2}\tan^2(\theta_0-\theta_1) & 0 \\ 0 & \frac{rI_{\theta_0}}{b} \end{bmatrix},$$
$$C_2(q,\dot{q}) = \begin{bmatrix} \frac{rI_{\theta_1}(\dot{\theta}_0-\dot{\theta}_1)\tan(\theta_0-\theta_1)}{d^2\cos^2(\theta_0-\theta_1)} & \frac{-ra_0m_0\dot{\theta}_0}{\cos(\theta_0-\theta_1)} \\ \frac{ra_0m_0\dot{\theta}_0}{b\cos(\theta_0-\theta_1)} & 0 \end{bmatrix},$$
$$I_{\theta_1} = m_1a_1^2 + m_0d^2 + I_1; \; I_{\theta_0} = m_0a_0^2 + I_0.$$

In this expression, m_0, m_1 and I_0, I_1 are mass and interia moment of Tractor and Trailer respectively. $M_2(q)$ is symmetric positive definite inertia matrix, $C_2(q,\dot{q})$ is centripetal, coriolis matrix.

Remark 1: By setting new control torque in (7) and multiplying both side of (6) by matrix $B_1^{-1}(q)$, we obtain the dynamic model (8) which is different from previous dynamic model in [7–9] by following property that unknown parameters $(a_0, a_1, m_0, m_1, I_{\theta_1}, I_{\theta_0})$ were put into matrix $M_2(q)$, $C_2(q,\dot{q})$.

2.2. Control objective

Let $(x_r, y_r, \theta_{1r}, \theta_{0r})$ denote the desired reference position and orientation of the virtual robot whose motion is satisfied:

$$\begin{aligned}
\dot{x}_r &= u_{1r}\cos\theta_{1r},\\
\dot{y}_r &= u_{1r}\sin\theta_{1r},\\
\dot{\theta}_{1r} &= u_{1r}d^{-1}\tan(\theta_{0r}-\theta_{1r}),\\
\dot{\theta}_{0r} &= u_{2r},
\end{aligned} \tag{9}$$

where u_{1r} and u_{2r} are the linear and angular velocities of virtual robot, respectively. Furthermore $(x_r(0), y_r(0))$ are initial position and $(\theta_{1r}(0), \theta_{0r}(0))$ are initial orientation of the reference trajectory.

The control objective is to find the control torque τ to apply to two wheels of Tractor to guarantee that the tracking errors between real trajectory and desired trajectory generated by (9) converge to the origin.

In order to achieve control objective, we give the following assumptions:

Assumption 1: All reference signals x_r, y_r, θ_{1r}, θ_{0r}, u_{1r}, u_{2r} and their first and second order time derivative are bounded, which are necessary to apply the Barbalat's lemma in [20] when proving the stability of the whole system with negative semi-definite derivative of the Lyapunov function in Theorem 1.

Assumption 2: $|u_{1r}(t)| \geq \mu\,\forall t\in[0,+\infty)$ where μ is strictly positive constant.

Assumption 3: The heading angles of virtual and real robot satisfy:

$$\begin{aligned}
&a)\ \ (\theta_{0r}-\theta_{1r}) \text{ and } (\theta_0-\theta_1) \neq k\pi+\frac{\pi}{2}\ \ \forall t\in[0,+\infty),\\
&b)\ \ (\theta_{1r}-\theta_1) \neq k\pi+\frac{\pi}{2}\ \ \forall t\in[0,+\infty).
\end{aligned}$$

Remark 2: The problem of the tracking smooth path continuously belongs to Assumption 2. In this case, the desired linear velocity is nonzero. The assumption 3a guarantees the existence of kinematic model (1) and virtual robot (9). The assumption 3b implies that the Trailer wheeled mobile robot cannot follow the desired orientation if it is perpendicular to current direction.

3. STATE FEEDBACK CONTROL DESIGN

Let us define tracking errors as follows:

$$\begin{bmatrix} e_x\\ e_y\\ e_{\theta_1}\\ e_{\theta_0}\end{bmatrix} = R\begin{bmatrix} x-x_r\\ y-y_r\\ \theta_1-\theta_{1r}\\ \theta_0-\theta_{0r}\end{bmatrix},$$

$$R=\begin{bmatrix}\cos(\theta_{1r}) & \sin(\theta_{1r}) & 0 & 0\\ -\sin(\theta_{1r}) & \cos(\theta_{1r}) & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}. \tag{10}$$

The convergence of $(e_x, e_y, e_{\theta_1}, e_{\theta_0})$ to the origin implies the achievement of the control objective. From kinematic model (1), the time – differentiating (10) is given as the following kinematic tracking errors:

$$\begin{aligned}
\dot{e}_x &= \dot{\theta}_{1r}e_y - u_{1r} + \cos(e_{\theta_1})u_1,\\
\dot{e}_y &= -\dot{\theta}_{1r}e_x + \sin(e_{\theta_1})u_1,\\
\dot{e}_{\theta_1} &= d^{-1}\tan(\theta_0-\theta_1)u_1 - \dot{\theta}_{1r},\\
\dot{e}_{\theta_0} &= -u_{2r}+u_2.
\end{aligned} \tag{11}$$

With the aid of transformation in [7], by setting:

$$\begin{aligned}
z_1 &= e_x,\\
z_2 &= e_y,\\
z_3 &= \tan(e_{\theta_1}),\\
z_4 &= \frac{\tan(\theta_0-\theta_1)-\tan(\theta_{0r}-\theta_{1r})\cos(e_{\theta_1})}{d\cos^3(e_{\theta_1})}+e_y,
\end{aligned} \tag{12}$$

$$\begin{aligned}
w_1 &= -u_{1r}+\cos(e_{\theta_1})u_1,\\
w_2 &= \dot{z}_4 = \beta_1u_1+\beta_2u_2+f_2,
\end{aligned} \tag{13}$$

where

$$\begin{aligned}
\beta_1 &= \frac{-\tan(\theta_{0r}-\theta_{1r})}{d^2\cos^3(e_{\theta_1})\cos^2(\theta_0-\theta_1)}+\sin(e_{\theta_1})\\
&\quad +3\frac{\tan^2(\theta_0-\theta_1)\sin(e_{\theta_1})}{d^2\cos^4(e_{\theta_1})}\\
&\quad -2\frac{\tan^2(\theta_{0r}-\theta_{1r})\sin(e_{\theta_1})}{d^2\cos^3(e_{\theta_1})},\\
f_2 &= -\dot{\theta}_{1r}\Bigg(\frac{3tan(\theta_0-\theta_1)\,sin(e_{\theta_1})}{dcos^4(e_{\theta_1})}+e_x\\
&\quad -\frac{2tan(\theta_{0r}-\theta_{1r})\,sin_{\theta_1}}{dcos^3(e_{\theta_1})}\Bigg)\\
&\quad -\frac{\dot{\theta}_{0r}-\dot{\theta}_{1r}}{d\cos^2(\theta_{0r}-\theta_{1r})\cos^2(e_{\theta_1})},\\
\beta_2 &= d^{-1}\cos^{-3}(e_{\theta_1})\cos^{-2}(\theta_0-\theta_1),\\
\Phi &= \begin{bmatrix}\cos(e_{\theta_1}) & 0\\ \beta_1 & \beta_2\end{bmatrix},\ f=\begin{bmatrix}-u_{1r} & f_2\end{bmatrix},\\
\omega &= \begin{bmatrix} w_1 & w_2\end{bmatrix}^T.
\end{aligned}$$

The input transformation (13) is rewritten as:

$$\omega = \Phi(\theta_0,\theta_1,\theta_{0r},\theta_{1r})u+f. \tag{14}$$

Equation (14) shows the input transformation which is not specified in [7]. By using (11), the time derivative both side of (12) is given as:

$$\begin{aligned}
\dot{z}_1 &= \dot{\theta}_{1r}z_2+w_1,\\
\dot{z}_2 &= -\dot{\theta}_{1r}z_1+(w_1+u_{1r})z_3,\\
\dot{z}_3 &= u_{1r}(z_4-z_2)
\end{aligned}$$

$$\begin{aligned}&+w_1\left(z_4-z_2+\tan\left(\theta_{0r}-\theta_{1r}\right)d^{-1}\left(1+z_3^2\right)\right),\\ \dot{z}_4 =& w_2.\end{aligned} \tag{15}$$

Now, we apply the Backstepping technique in [19] to design $\bar{\tau}$. Defining virtual control errors $\tilde{w}_1$ and $\tilde{w}_2$ as:

$$\tilde{w}_1 = w_1 - w_{1c},\ \ \tilde{w}_2 = w_2 - w_{2c}, \tag{16}$$

where w_{1c} and w_{2c} are virtual controls for w_1 and w_2, substituting (16) into (15):

$$\begin{aligned}\dot{z}_1 =& \dot{\theta}_{1r}z_2 + w_{1c} + \tilde{w}_1,\\ \dot{z}_2 =& -\dot{\theta}_{1r}z_1 + \left(w_{1c}+\tilde{w}_1+u_{1r}\right)z_3,\\ \dot{z}_3 =& u_{1r}\left(z_4-z_2\right) + \left(w_{1c}+\tilde{w}_1\right)\left(z_4-z_2\right.\\ &\left.+\tan\left(\theta_{0r}-\theta_{1r}\right)d^{-1}\left(1+z_3^2\right)\right),\\ \dot{z}_4 =& w_{2c}+\tilde{w}_2/\end{aligned} \tag{17}$$

In order to design w_{1c} and w_{2c}, the first Lyapunov candidate function is selected as:

$$V_1 = \frac{1}{2}\left(z_1^2+z_2^2+z_3^2+\frac{1}{k}z_4^2\right), \tag{18}$$

where k is a positive constant, differentiating both sides of (18) along the solution of (17):

$$\begin{aligned}\dot{V}_1 =& z_4\left(z_3u_{1r}+\frac{1}{k}w_{2c}\right)+z_1w_{1c}\\ &+z_3w_{1c}\left(z_4+\frac{\tan\left(\theta_{0r}-\theta_{1r}\right)}{d}\left(1+z_3^2\right)\right)+\frac{1}{k}z_4\tilde{w}_2\\ &+z_1\tilde{w}_1+z_3\tilde{w}_1\left(z_4+\frac{\tan\left(\theta_{0r}-\theta_{1r}\right)}{d}\left(1+z_3^2\right)\right),\end{aligned} \tag{19}$$

and selecting w_{1c}, w_{2c} as follows:

$$\begin{aligned}w_{1c} =& -k_2z_3\left(z_4+\frac{\tan\left(\theta_{0r}-\theta_{1r}\right)}{d}\left(1+z_3^2\right)\right)\\ &-k_2z_1,\\ w_{2c} =& k\left(-z_3u_{1r}-k_1z_4\right).\end{aligned} \tag{20}$$

It is clear that (19) becomes:

$$\dot{V}_1 = -k_1z_4^2+\varpi^T\Omega-k_2{\Omega_1}^2, \tag{21}$$

where

$$\begin{aligned}&\varpi = \begin{bmatrix} \tilde{w}_1 & \tilde{w}_{\ 2}\end{bmatrix}^T;\ \ \Omega = \begin{bmatrix}\Omega_1 & \Omega_2\end{bmatrix}^T,\\ &\Omega_1 = z_1+z_3\left(z_4+\frac{\tan\left(\theta_{0r}-\theta_{1r}\right)}{d}\left(1+z_3^2\right)\right),\\ &\Omega_2 = \frac{1}{k}z_4.\end{aligned}$$

Futher, by setting:

$$u_c = \Phi^{-1}\left(\omega_c - f\right), \tag{22}$$

where $\omega_c = \begin{bmatrix} w_{1c} & w_{2c}\end{bmatrix}^T$, combining (14) and (22), provides:

$$\varpi = \Phi\left(u-u_c\right) = \Phi\tilde{u}. \tag{23}$$

The design of the control torque $\bar{\tau}$ deduced from taking the second Lyapunov candidate function:

$$V_2 = V_1 + \frac{1}{2}\tilde{u}^TM_2\left(q\right)\tilde{u}. \tag{24}$$

Time differentiating both side of (24):

$$\begin{aligned}\dot{V}_2 =& -k_1z_4^2-k_2\Omega_1^2+\varpi^T\Omega\\ &+\tilde{u}^TM_2\left(q\right)\left(\dot{u}-\dot{u}_c\right)+\frac{1}{2}\tilde{u}^T\dot{M}_2\left(q\right)\tilde{u}.\end{aligned} \tag{25}$$

Substituting (23) and (8) into (25):

$$\begin{aligned}\dot{V}_2 =& -k_1z_4^2-k_2\Omega_1^2+\tilde{u}^T\Big[\bar{\tau}-C_2\left(q,\dot{q}\right)u\\ &-M_2\left(q\right)\dot{u}_c+\frac{1}{2}\dot{M}_2\left(q\right)\tilde{u}+\Phi^T\Omega\Big].\end{aligned} \tag{26}$$

Parameterization of terms in the last bracket of (26):

$$C_2\left(q,\dot{q}\right)u+M_2\left(q\right)\dot{u}_c-\frac{1}{2}\dot{M}_2\left(q\right)\tilde{u} = Y\theta, \tag{27}$$

where

$$Y = \begin{bmatrix} \Xi & 0\\ \dfrac{\dot{\theta}_0u_2}{\cos\left(\theta_0-\theta_1\right)} & 0\\ \dot{u}_{1c} & 0\\ 0 & \dfrac{\dot{\theta}_0u_1}{\cos\left(\theta_0-\theta_1\right)}\\ 0 & \dot{u}_{2c}\end{bmatrix}^T,$$

$$\Xi = \frac{\left(\dot{\theta}_0-\dot{\theta}_1\right)\tan\left(\theta_0-\theta_1\right)u_{1c}}{\cos^2\left(\theta_0-\theta_1\right)}+\tan^2\left(\theta_0-\theta_1\right)\dot{u}_{2c},$$

$$\theta = \begin{bmatrix}\dfrac{rI_{\theta_1}}{d^2} & ra_0m_0 & \left(m_0+m_1\right)r & \dfrac{ra_0m_0}{b} & \dfrac{rI_{\theta_0}}{b}\end{bmatrix}^T.$$

Selecting $\bar{\tau}$ as:

$$\bar{\tau} = Y\hat{\theta}-\Gamma\tilde{u}-\Phi^T\Omega, \tag{28}$$

where Γ is positive defined matrix, the equation (26) becomes:

$$\dot{V}_2 = -k_1z_4^2-k_2\Omega_1^2-\tilde{u}^T\Gamma\tilde{u}-\tilde{u}^TY\tilde{\theta}. \tag{29}$$

The adaptive law is designed by choosing the third Lyapunov function candidate:

$$V_3 = V_2+\frac{1}{2}\tilde{\theta}^T\Lambda\tilde{\theta}, \tag{30}$$

where $\Lambda = \Lambda^T$ is positive defined matrix. The proposed adaptive law is:

$$\dot{\hat{\theta}} = -\Lambda^{-1}Y^T\tilde{u}. \tag{31}$$

Applying (31) and differentiating both side of (30):

$$\dot{V}_3 = -k_1z_4^2-\tilde{u}^T\Gamma\tilde{u}-k_2\Omega_1^2. \tag{32}$$

Lemma 1 [6]**:** For any differentiable function $f : R^+ \to R$. If $f(t)$ converges to zero as $t \to +\infty$ and its derivative satisfies:

$$\dot{f}(t) = f_0(t) + \eta(t), \forall t > 0,$$

where f_0 is a uniformly continuous function and $\eta(t)$ tends to zero as $t \to +\infty$, $\dot{f}(t)$ and $f_0(t)$ tend to zeros as $t \to +\infty$.

Theorem 1: Under Assumptions 1, 2 and 3, there exist the positive control parameters k, k_1, k_2, Γ such that the controller (28) and the adaptive law (31) force the Tractor-Trailer (1) (6) to asymptotically track the virtual robot (9).

Proof: By selecting positive constants k, k_1, k_2 and positive define matrix Γ, the time derivative of Lyapunov function in (32) is negative semi-define, which implies that the Lyapunov funtion V_3 is bounded and z_1, z_2, z_3, z_4, $\tilde{u}_1$, $\tilde{u}_2$ consequently are bounded. Under assumption 1, from dynamic system (17), it can be derived the boundedness of $\dot{z}_1$, $\dot{z}_2$, $\dot{z}_3$, $\dot{z}_4$, $\dot{\tilde{u}}_1$, $\dot{\tilde{u}}_2$. Therefore, we apply Barbalat's lemma in Theorem 8.4 of [20], the time $-$ derivative (32) tends to the origin as time to infinity:

$$\lim_{t\to+\infty} z_4 = 0, \quad \lim_{t\to+\infty} \Omega_1 = 0, \quad \lim_{t\to+\infty} \tilde{u} = 0. \tag{33}$$

Under Assumption 3 and matrix Φ is invertible, according to the definition (23):

$$\lim_{t\to+\infty} (\tilde{w}_1, \tilde{w}_2) = 0. \tag{34}$$

Substituting (20) into (17). Then, we have:

$$\frac{d(z_4)}{dt} = -ku_{1r}z_3 - kk_1z_4 + \tilde{w}_2. \tag{35}$$

Hence, the use of Lemma 1 offers:

$$\lim_{t\to+\infty} \frac{d(z_4)}{dt} = 0, \lim_{t\to+\infty} z_3 = 0. \tag{36}$$

From (33) and (36), because z_3, z_4 go to zero as $t \to +\infty$, we have $\lim_{t\to+\infty} z_1 = 0$:

$$\frac{d(z_3)}{dt} = (w_{1c} + \tilde{w}_1)\left(z_4 - z_2 + \frac{\tan(\theta_{0r} - \theta_{1r})}{d}(1 + z_3^2)\right) - u_{1r}z_2 + u_{1r}z_4.$$

Because of $\lim_{t\to+\infty}(z_1, z_3, z_4) = 0$ so $\lim_{t\to+\infty} w_{1c} = 0$. Applying lemma 1 again, we have that z_2 go to the origin as $t \to +\infty$. □

Remark 3: The previous paper [4] applied coordinate change to angular variable error of one wheeled mobile robot. Because tracking control design for two wheeled mobile robot is more complicated, authors inherit coordinate change to all four variable error in [7], but proposing separate control input as (13) and (14) to show the relationship between virtual control of kinematics and control input of dynamics. The Lyapunov function V_2 (24) including two parts which are V_1 of kinematics and control input error of dynamics indicates the stability of the whole system by using Barbalat's lemma in [20], which was ignored in [7].

4. SIMULATIONS

In order to verify the proposed method in this paper, we show the simulation results which is desired tracking trajectory of Tractor Trailer Wheeled Mobile Robot. The parameters of the proposed controller are given as : $m_0 = 1$ (kg), $m_1 = 0.4$ (kg), $I_1 = 0.005$ (kg.m^2), $I_0 = 0.002$ (kg.m^2), $d = 0.15$ (m), $a_0 = 0.03$ (m), $r = 0.03$ (m), $b = 0.06$ (m) and the initial states of system is chosen such that initial position of mobile robot is out of reference trajectories: $u(0) = [0;0]$, $x(0) = 1.5$ (m), $y(0) = 0.5$ (m), $\theta_1(0) = -0.1$ (rad), $\theta_0(0) = -0.2$ (rad), $x_r(0) = 2$ (m), $y_r(0) = 0$ (m), $\theta_{1r} = 0$ (rad), $\theta_{0r}(0) = 0$ (rad), $\hat{\theta}(0) = [0.02, 0.02, 0.3, 0.3, 0.01]^T$. It is based on Theorem 1, all coefficients are selected as follows: $\Lambda = 0.1\text{diag}([1,1,1,1])$, $k_1 = 0.6$, $k_2 = 25$, $\Gamma = 5\text{diag}([1,1])$, $k = 100$.

Simulation scenario is that the considered position of trailer at initial state is out of the reference trajectory. Therefore, the control objective is that the proposed controllers and the proposed adaptive law make system enable to track this given trajectory with finite time simulation. The polygon reference trajectories have some angles which do not satisfy the Assumption 3, therefore, we have to create some smooth angles as described in the Figs. 7-9.

The desired η polygon trajectory is designed in two dimensional space as follows:

$$\begin{aligned} x_r &= a[R + \cos(\eta\alpha t)]\cos(\alpha t), \\ y_r &= a[R + \cos(\eta\alpha t)]\sin(\alpha t), \end{aligned} \tag{37}$$

where a and α is the scale and the angular velocity of reference trajectory, respectively and R is the radius of the reference trajectory. It is selected as $a = 0.1$, $\alpha = 0.5$ (rad/s) and $R = 2$ (m).

This desired trajectory (37) combines with virtual robot (9) to generate linear and angular velocity:

$$\begin{aligned} u_{1r} &= \sqrt{(\eta a\alpha)^2\sin^2(\eta\alpha t) + (a\alpha)^2[R + \cos^2(\eta\alpha t)]^2}, \\ u_{2r} &= \frac{d_1(\ddot{u}_{1r}\dot{x}_r + \dot{u}_{1r}\ddot{x}_r - {}_r u_{1r} - \ddot{x}_r\dot{u}_{1r})\dot{y}_r u_{1r}^2}{\dot{y}_r^2 u_{1r}^4 + d_1^2(\dot{u}_{1r}\dot{x}_r - \ddot{x}_r u_{1r}^2)^2} \\ &\quad - \frac{d_1(\dot{u}_{1r}\dot{x}_r - \ddot{x}_r u_{1r})(\ddot{y}_r u_{1r}^2 + 2\dot{y}_r u_{1r}\dot{u}_{1r})}{\dot{y}_r^2 u_{1r}^4 + d_1^2(\dot{u}_{1r}\dot{x}_r - \ddot{x}_r u_{1r}^2)^2}. \end{aligned} \tag{38}$$

As shown in Figs. 2-3, the position and angular error tend to zero as time go to infinity. Figs. 4-5 present

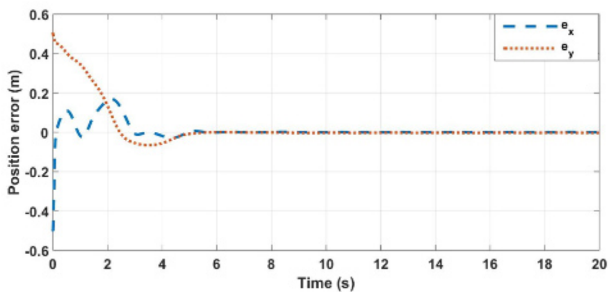

Fig. 2. The position error.

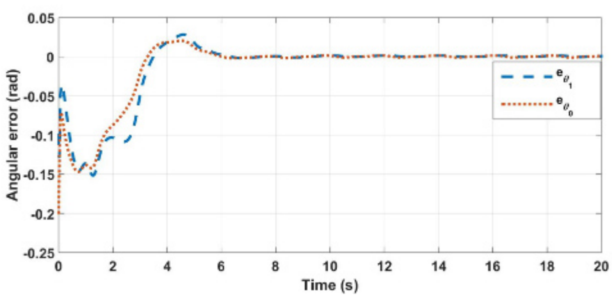

Fig. 3. The angular error.

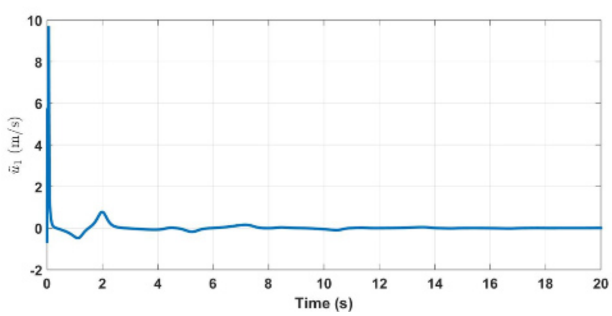

Fig. 4. The linear velocity error.

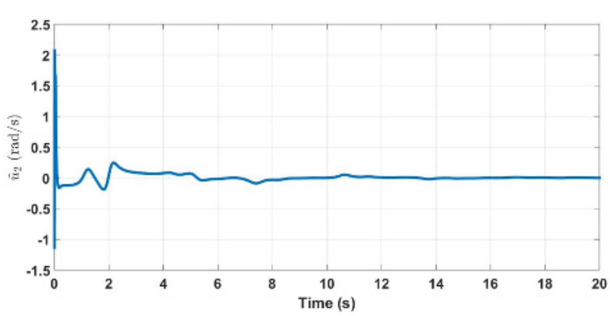

Fig. 5. The angular velocity error.

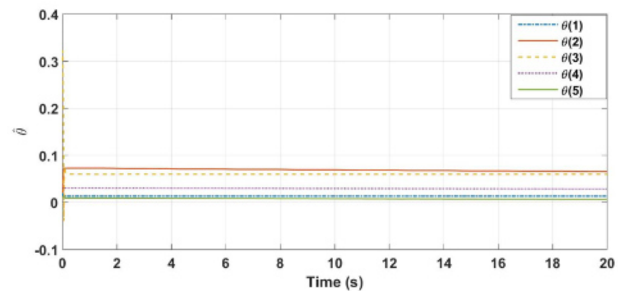

Fig. 6. The estimated parameters.

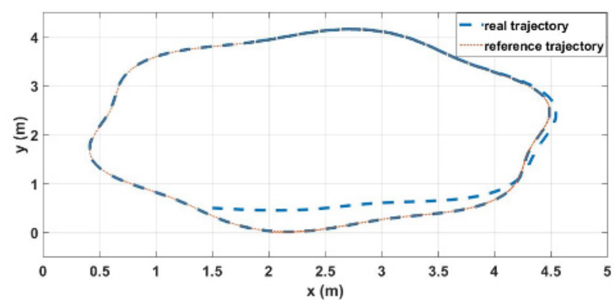

Fig. 7. Desire trajectory as 6 sided smooth polygon.

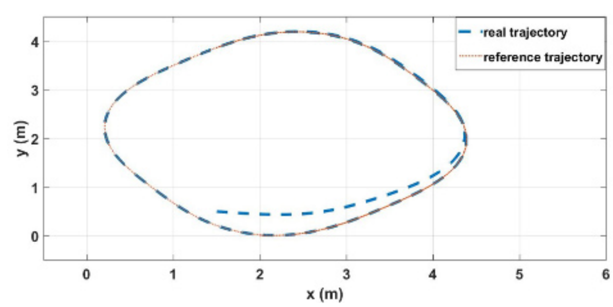

Fig. 8. Desire trajectory as 4 sided smooth polygon.

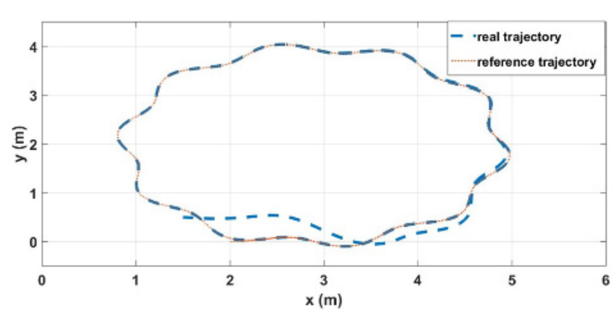

Fig. 9. Desire trajectory as 10 sided smooth polygon.

the zero convergence of linear and angular velocity error, which is consistent with Theorem 1. In addition, Fig. 6 shows the boundedness of estimated parameters. The dashed line in Fig. 7 stands for the polygon trajectories on which, the considered TTWMR system attempt to track with different initial states. Through several built model trajectories in Figs. 8-9, it is clear that the demonstrated method brings the good performance and effectiveness in tracking control.

5. CONCLUSION

This paper has presented the time varying adaptive controller which has been proposed to carry out tracking control for the Tractor Trailer Wheeled Mobile Robot. The main success of this paper is modification of using Backstepping technique from kinematics to dynamics. The simulation results for several cases of trajectory demonstrate the effective performance of the proposed method to trajectory tracking control. In future work, we will apply this proposed approach to extend to nonholomic systems including a Tractor and several Trailers.

REFERENCES

[1] K. D. Do, Z. P. Jiang, and J. Pan, "Simultaneous tracking and stabilization of mobile robots: an adaptive approach," *IEEE Trans. on Automatic Control*, vol. 49, no. 7, pp. 1147-1152, 2004.

[2] T. Fukao, H. Nakagawa, and N. Adachi, "Adaptive tracking control of a nonholonomic mobile robot," *IEEE Trans. on Robotics and Automation* , vol. 16, no. 5, pp. 609-615, 2000.

[3] S. S. Ge, Z. Wang, and T. H. Lee, "Adaptive stabilization of uncertain nonholonomic systems by state and output feedback," *Automatica*, vol. 39, no. 8, pp. 1451-1460, 2003.

[4] J. Huang, C. Wen, W. Wang, and Z. P. Jiang, "Adaptive output feedback tracking control of a nonholonomic mobile robot," *Automatica*, vol. 50, no. 3, pp. 821-831, 2014.

[5] R. M. Murray and S. S. Sastry, "Nonholonomic motion planning: steering using sinusoids," *IEEE Trans. on Automatic Control*, vol. 39, no. 5, pp. 700-716, 1993.

[6] Z. P. Jiang and H. Nijmeijer, "A recursive technique for tracking control of nonholonomic systems in chained form," *IEEE Trans. on Automatic Control*, vol. 44, no. 2, pp. 265-279, 1999.

[7] A. K. Khalaji and S. A. A. Moosavian, "Robust adaptive controller for a tractor-trailer mobile robot," *IEEE/ASME Trans. on Mechatronics*, vol. 19, no. 3, pp. 943-953, 2013.

[8] E. Kayacan and E. Kayacan, "Robust tube-based decentralized nonlinear model predictive control of an autonomous tractor-trailer system, " *IEEE/ASME Trans. on Mechatronics*, vol. 20, no. 1, pp. 447-456, 2014.

[9] E. Kayacan, E. Kayacan, H. Ramon, and W. Saeys, "Learning in centralized nonlinear model predictive control: application to an autonomous tractor-trailer system," *IEEE Trans. on Control Systems Technology*, vol. 23, no. 1, pp. 197-205, 2014.

[10] R. Fierro and F. L. Lewis, "Control of a nonholonomic mobile robot: backstepping kinematics into dynamics," *Proc. of the 34th IEEE Conference on Decision and Control*, pp. 3805-3810, 1995.

[11] W. He, Y. Dong, and C. Sun, "Adaptive neural impedance control of a robotic manipulator with input saturation, " *IEEE Trans. on Systems, Man, and Cybernetics: Systems*, vol. 46, no. 3, pp. 334-344, 2016.

[12] W. He, Y. Chen, and Z. Yin, "Adaptive neural network control of an uncertain robot with full-state constraints," *IEEE Trans. on Cybernetics*, vol. 46, no. 3, pp. 620-629, 2016.

[13] W. He and Y. Dong, "Adaptive fuzzy neural network control for a constrained robot using impedance learning," *IEEE Trans. on Neural Networks and Learning Systems*, vol. 29, no. 4, pp. 1174-1186, 2018.

[14] W. He, Z. Yan, C. Sun, and Y. Chen, "Adaptive neural network control of a flapping wing micro aerial vehicle with disturbance observer," *IEEE Trans. on Cybernetics*, vol. 47, no. 10, pp. 3542-3465, 2017.

[15] D.-P. Li, Y.-J. Liu, S. Tong, C. L. P. Chen, and D.-J. Li, "Neural networks-based adaptive control for nonlinear state constrained systems with input delay," *IEEE Trans. on Cybernetics*, to be published. DOI: 10.1109/TCYB.2018.2799683

[16] Y. J. Liu and S. Tong, "Barrier Lyapunov functions for Nussbaum gain adaptive control of full state constrained nonlinear systems, " *Automatica*, vol. 76, pp. 143-152, 2017.

[17] Y. J. Liu, S. Lu, S. Tong, X. Chen, C. L. P. Chen, and D. J. Li, "Adaptive control-based Barrier Lyapunov Functions for a class of stochastic nonlinear systems with full state constraints," *Automatica*, vol. 87, pp. 83-93, 2018.

[18] Y. J. Liu, M. Gong, S. Tong, C. L. P. Chen, and D. J. Li, "Adaptive fuzzy output feedback control for a class of nonlinear systems with full state constraints," *IEEE Trans. on Fuzzy Systems*, vol. 26, no. 5, pp. 2607-2617, 2018.

[19] M. Krstic, I. Kanellakopoulos, and P. V. Kokotovic, *Nonlinear and Adaptive Control Design*, Wiley, New York, 1995.

[20] H. K. Khalil, *Nonlinear Systems*, 3rd ed., Prentice-Hall, NJ, 2002.

Thanh Binh Nguyen received the B.S. and M.S. degrees in Electrical Engineering from Hanoi University of Science and Technology, Hanoi, Vietnam, in 2014 and 2016, respectively. He helds the position as lecturer at Thuyloi University, Vietnam in 2017. Currently, he is a Ph.D. student in University of Ulsan, Korea. His research interests include control of air, land and underwater vehicles, nonlinear and robust adaptive control.

Nguyen Anh Tung is a senior student of talented program at Hanoi University of Science and Technology, Hanoi, Vietnam. He received HONDA Young Engineer and Scientist award 2017 for TOP 10 excellent student in Vietnam. His research interests include optimal control, nonlinear control and adaptive control.

Dao Phuong Nam received the Ph.D. degree in Electrical Engineering from Hanoi University of Science and Technology, Hanoi, Vietnam in 2013. Currently, he holds the position as lecturer at Hanoi University of Science and Technology, Vietnam. His research interests include control of robotic systems and robust/adaptive, optimal control.

Nguyen Hong Quang received his B.S. and M.S. degrees in electrical engineering from Thainguyen University of technology (TNUT) and Hanoi University of Science and Technology (HUST), Vietnam, in 2007 and 2012, respectively. He is currently with TNUT as Ph.D. student and Lecturer. His research interests are control of electrical drive systems and robust nonlinear model predictive control.

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.