A Zero-Sum Game Framework for Optimal Sensor Placement in Uncertain Networked Control Systems under Cyber-Attacks

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Swedish Research Council

Stiftelsen för Strategisk Forskning

Outline

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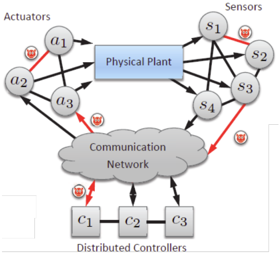
Motivation



Several cyber incidents on Cyber-physical systems in the past

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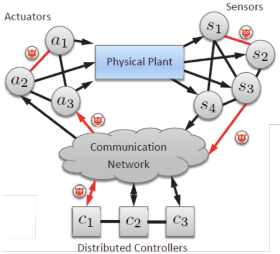

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1. DoS attack on the Ukrainian power grid in 2015.
2. Data injection attack on Kemuri water distribution company in 2016.
3. . . . and many more.

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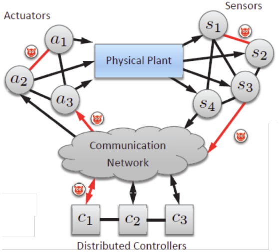

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Lesson: Be proactive and protect the system, even uncertain system modeling

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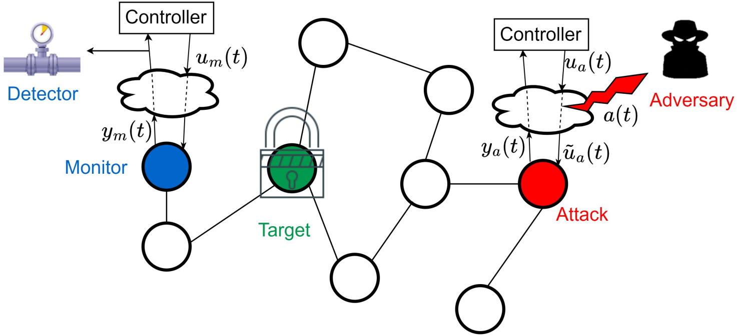
Problem description

The main research question

Given an uncertain networked control system (multi-agent system) under cyber-attacks, how to place a sensor at an agent s.t. minimizing the risk on a given local performance.



System description

- Undirected connected graph $\mathcal{G}$ with N agents

$$\dot{x}_i^{\Delta}(t) = \sum_{v_j \in \mathcal{N}_i} \ell_{ij}^{\Delta}\big(x_i^{\Delta}(t) - x_j^{\Delta}(t)\big) + \tilde{u}_i(t),\ \ v_i \in \big\{v_1, v_2, \ldots, v_N\big\},$$

$$y_{\tau}^{\Delta}(t) = x_{\tau}^{\Delta}(t),$$

- Healthy/Attacked local controller

$$\tilde{u}_i(t) = -\theta_i^{\Delta} x_i^{\Delta}(t) + \begin{cases} 0, & \text{if } v_i \text{ is healthy} \\ a(t), & \text{if } v_i \text{ is attacked} \end{cases}$$

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- Healthy closed-loop model ($a(t) = 0$)

$$\dot{x}^{\Delta}(t) = -L^{\Delta} x^{\Delta}(t),\ \ L^{\Delta} \triangleq \bar{L} + \Delta,\ \ \Delta \in \Omega.$$

Assume that Ω is a compact set.

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Detector and Adversary description

- **Revisit**: Undirected connected graph $\mathcal{G}$ (vertex set $\mathcal{V}$, edge set $\mathcal{E}$, L^{Δ}), protected target vertex v_τ, and closed-loop healthy system

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• **Prior information**
Know: Sets $\mathcal{V}$, $\mathcal{E}$, Ω; location v_{τ}; nominal $\bar{L}$.
Don't know: Δ; their rivals' exact actions.

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Detector and Adversary Purpose

- Fixed local performance at protected vertex v_τ: $\|y_\tau\|^2_{\mathcal{L}_2[0,T]}$

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Choose $v_a \in \mathcal{V} \setminus \{v_\tau\}$; design $a(t)$ as **stealthy** $\|y_m^\Delta\|^2_{\mathcal{L}_2[0,T]} \leq \sigma$

And maximize $\|y_\tau\|^2_{\mathcal{L}_2[0,T]}$ where $\tilde{u}_a(t) = u_a(t) + a(t)$

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- Detector purpose:

Choose $v_m \in \mathcal{V} \setminus \{v_\tau\}$ to detect cyber-attack $\|y_m^\Delta\|^2_{\mathcal{L}_2[0,T]} > \sigma$

And $\|y_\tau\|^2_{\mathcal{L}_2[0,T]}$ as low as possible

Outline

Risk on local performance

- Worst-case attack impact on local performance

$$\sup_{a \in \mathcal{L}_2[0,T]} J_\tau(v_a, v_m; \Delta, a),$$

$$J_\tau(v_a, v_m; \Delta, a) \triangleq \left\| y_\tau^\Delta \right\|^2_{\mathcal{L}_2[0,T]} \mathbb{I}_{\mathcal{A}}(a),$$

$$\mathcal{A} \triangleq \{a | \ \left\| y_m^\Delta \right\|^2_{\mathcal{L}_2[0,T]} \leq \sigma, \ x(0) = 0\},$$

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- Risk metric - Value at Risk (VaR_β) over uncertainty set Ω

$$\mathcal{J}_\tau(v_a, v_m) = \text{VaR}_{\beta,\Omega}\Big[\sup_{a \in \mathcal{L}_2[0,T]} J_\tau(v_a, v_m; \Delta, a)\Big]$$

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Illustrate VaR →

Value at Risk

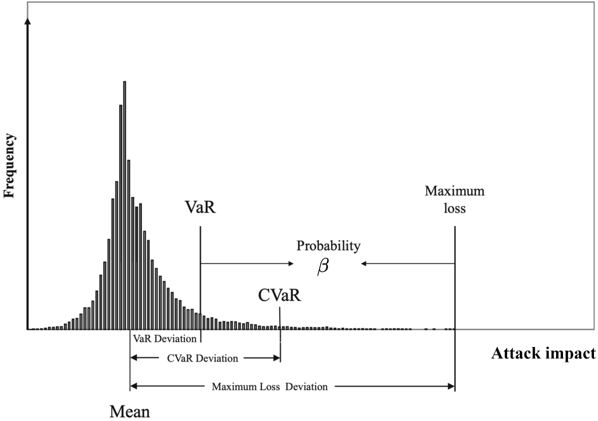

Figure: Risk metrics

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Problem formulation

Problem formulation (zero-sum game)

Given protected target vertex v_τ, game payoff $\mathcal{J}_\tau(v_a, v_m)$

$$\min_{v_m \neq v_\tau \in \mathcal{V}} \; \max_{v_a \neq v_\tau \in \mathcal{V}} \; \mathcal{J}_\tau(v_a, v_m).$$

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The detector and the adversary satisfy[1]

$$-\infty < \mathcal{J}_\tau(v_a, v_m^\star) \leq \mathcal{J}_\tau(v_a^\star, v_m^\star) \leq \mathcal{J}_\tau(v_a^\star, v_m) < \infty, \quad \forall v_a, v_m \in \mathcal{V} \setminus \{v_\tau\}.$$

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Approximating game payoff

- Uncertainty set Ω, take M_1 sampled uncertainty values $\Delta_i \in \Omega,\ i = \{1, 2, \dots, M_1\}$

Theorem 4.1

Let $\epsilon_1, \beta_1 \in (0, 1)$ be chosen such that

$$\mathbb{P}\{|\mathbb{P}_\Omega(X < \gamma) - \hat{\mathbb{P}}_{M_1}| > \epsilon_1\} \leq \beta_1$$

where $\hat{\mathbb{P}}_{M_1} \triangleq \frac{1}{M_1} \sum_{i=1}^{M_1} \mathbb{I}\left(X \leq \gamma\right)$, where $M_1 \geq \frac{1}{2\epsilon_1^2} \log \frac{2}{\beta_1}$.
Then, VaR_β with an accuracy ϵ_1 and confidence β_1 by

$$\hat{\gamma} \triangleq \min \ \gamma \quad \text{s.t.} \ \hat{\mathbb{P}}_{M_1} \geq 1 - \beta.$$

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- Q: Should we evaluate M_1 game payoff values?

Evaluating game payoff

- We only need to evaluate $\lceil M_1(1-\beta_1)\rceil$ values *(Lemma 4.2)*

E.g., $\epsilon_1 = 0.06$, $\beta_1 = 0.08$, $M_1 \geq 450 \Rightarrow$ evaluate 414 values

[2] Ferrari, R. M., & Teixeira, A. M. (Eds.). (2021). Safety, Security and Privacy for Cyber-Physical Systems. Cham: Springer.

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- Worst-case attack impact with a sampled uncertainty Δ_i.

$$\gamma_i^\star \triangleq \sup_{a \in \mathcal{L}_2[0,T]} \left\| y_T^{\Delta_i} \right\|^2 \quad \text{s.t.} \quad \left\| y_m^{\Delta_i} \right\|^2 \leq \sigma$$

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- Solved via LMIs[2]

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- Solved via LMIs[2]
- Always have $\gamma_i^\star < \infty$?

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Problem Description

Problem Formulation

Proposed method

Numerical examples

Conclusions

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- Solved via LMIs[2]
- Always have $\gamma_i^\star < \infty$?
- Invariant zeros[3] of $\Sigma_m = (-L^{\Delta_i}, e_a, e_m^\top, 0)$ where $y_m^{\Delta_i}(t)$ is its output: unstable finite and infinite

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Invariant zeros

- Consider invariant zeros of $\Sigma_m = (A, B, C_m, 0)$ where $y_m(t)$ is its output.

$$\begin{bmatrix} \lambda I - A & -B \\ C_m & 0 \end{bmatrix} \begin{bmatrix} \bar{x} \\ g \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \bar{x} \neq 0. \tag{1}$$

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Lemma 4.4 (choice of parameters)

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- **Infinite invariant zeros** $\lambda = 1/s$ **where** $s = 0$ **satisfies (1)**

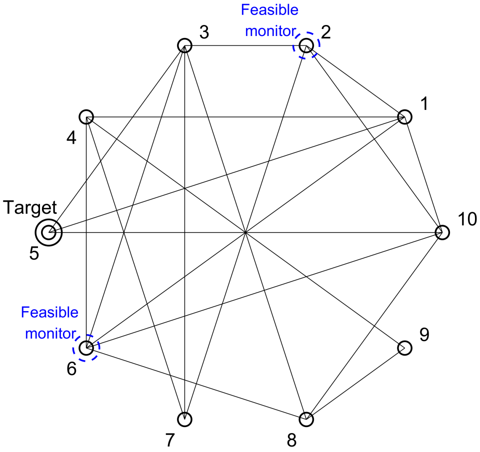
Relative degree r_Σ of a linear system Σ

Σ_m has output $y_m(t)$ and Σ_τ has output $y_\tau(t)$

$$r_{\ \Sigma_m} \leq r_{\ \Sigma_\tau}$$

Outline

Numerical examples



Two cases of Value-at-Risk VaR_β where

1) $\beta = 0.08$
2) $\beta = 0.15$

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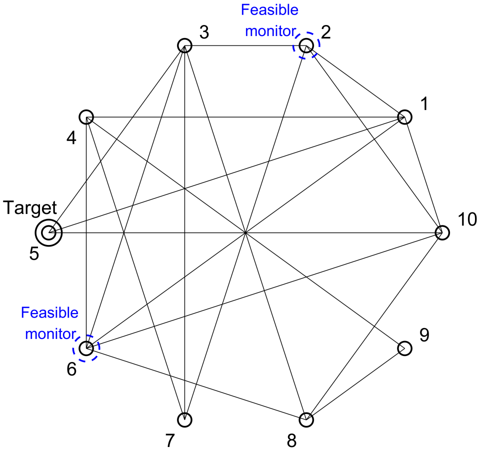

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What are the best choices for the detector and the adversary?

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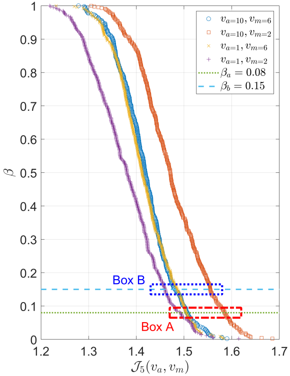
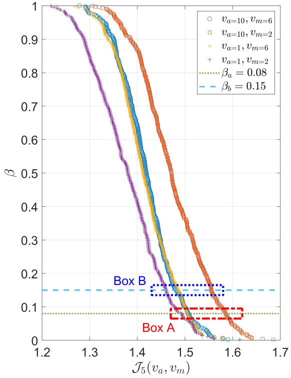


- Case 1 (Box A): $\beta = 0.08$

$$\begin{aligned} &\mathcal{J}_5(\forall v_a \in \mathcal{V} \setminus \{v_5, v_{10}\}, v_{m=6}) \\ &< \mathcal{J}_5(v_{a=10}, v_{m=6}) \\ &< \mathcal{J}_5(v_{a=10}, v_{m=2}). \end{aligned}$$

Numerical examples



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- Case 2 (Box B): $\beta = 0.15$

$$\mathcal{J}_5(v_{a=1}, v_{m=2}) = 1.4603,$$
$$\mathcal{J}_5(v_{a=10}, v_{m=6}) = 1.4803,$$
$$\mathcal{J}_5(v_{a=1}, v_{m=6}) = 1.4856,$$
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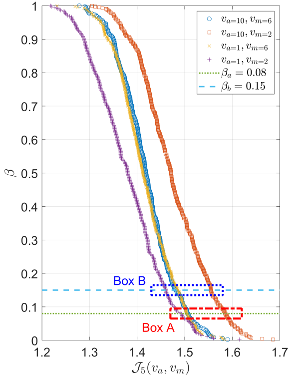

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$$\mathcal{J}_5(v_{a=10}, v_{m=2}) = 1.5550.$$

$$\mathbb{P}^\star(v_{m=6}) \approx 94.72\%, \ \mathbb{P}^\star(v_{m=2}) \approx 5.28\%,$$
$$\mathbb{P}^\star(v_{a=10}) \approx 25.29\%, \ \mathbb{P}^\star(v_{a=1}) \approx 74.71\%,$$
$$\mathbb{P}^\star(\forall v_a \in \mathcal{V} \setminus \{1, 5, 10\}) = 0\%.$$

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Conclusions

- We considered uncertain networked control systems under cyber-attacks
- The problem was formulated through zero-sum game framework
- We evaluated and computed the risk to find optimal sensor placement
- We illustrated the proposed method through a numerical example

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Questions!!!