Optimal Detector Placement in Networked Control Systems under Cyber-attacks with Applications to Power Networks

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IFAC World Congress
Yokohama, Japan, July 2023

Swedish Research Council

Stiftelsen för Strategisk Forskning

Outline

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Motivation

Problem Formulation

Optimal Detector Placement

A case study

Conclusion and future work

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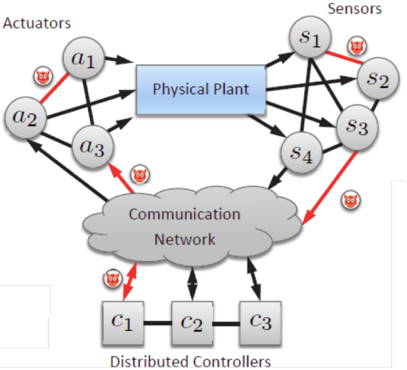

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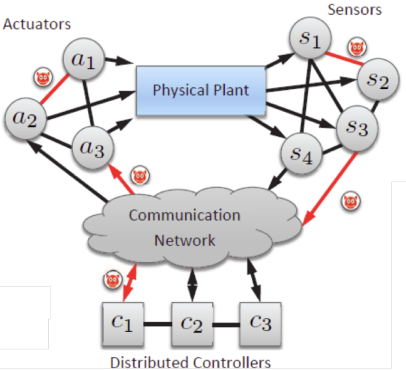

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1. DoS attack on the Ukrainian power grid in 2015.
2. Data injection attack on Kemuri water distribution company in 2016 . . . and more.

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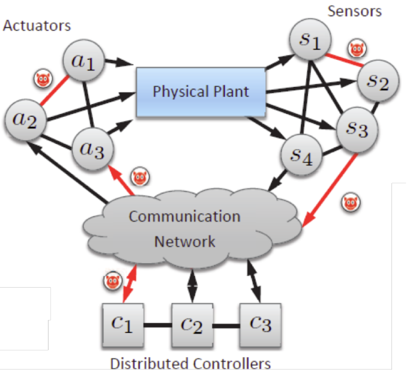

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Lesson: Be proactive and protect the system.

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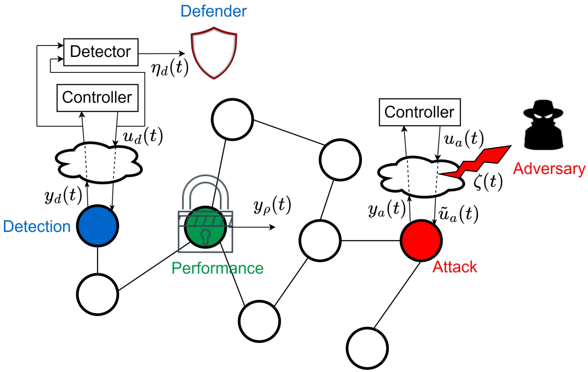
Problem description

The main research question

Given a networked control system (multi-agent system) under stealthy attacks, which detector should be monitored to minimize the risk on a given local performance.



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System Description

- **Main focus:** Power networks by linearized swing equations

$$m_i\ddot{p}_i(t) + h_i\dot{p}_i(t) = \sum_{j\in\mathcal{N}_i} \ell_{ij}\Big(p_i(t) - p_j(t)\Big) + \tilde{u}_i(t),$$

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- Control input under attacks

$$\tilde{u}_i(t) = u_i(t) + \begin{cases} 0, & i \in \mathcal{V}_{-a}, \\ \zeta(t), & i \equiv a \end{cases}$$

- Healthy $u_i(t)$ is designed s.t. $p_i(t),\ \dot{p}_i(t) \to 0$ (*Lemma 1*)

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Assumption: The entire network is at its equilibrium $(p_e = 0,\ \dot{p}_e = 0)$ before being attacked.

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System Description (Cont.)

- Network under cyber-attacks

$$\dot{x}(t) = Ax(t) + E_a\zeta(t),$$
$$y_i(t) = C_i x(t), \quad \forall i \in \mathcal{V},$$
$$y_\rho(t) = C_\rho x(t),$$

- Local performance: $\|y_\rho\|^2_{\mathcal{L}_2[0,T]} = \frac{1}{T}\int_0^T |y_\rho(t)|^2 \, \mathrm{d}t$

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- The defender monitors $\|\eta_d\|^2_{\mathcal{L}_2[0,T]} = \frac{1}{T}\int_0^T |\eta_d(t)|^2 \, \mathrm{d}t$

Resources and Strategies

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Location of performance ρ, the appearance of competitors, system parameters, and the detection mechanism

- **Defense strategy**: Select agent d and monitor $\|\eta_d\|^2_{\mathcal{L}_2[0,T]}$ such that minimizing the disruption $\|y_\rho\|^2_{\mathcal{L}_2[0,T]}$

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- **Attack policy**: Select agent a and design stealthy attack $\zeta(t)$ such that
  1) be stealthy $\|\eta_d\|^2_{\mathcal{L}_2[0,T]} \leq \delta^2$; and
  2) maximize the disruption $\|y_\rho\|^2_{\mathcal{L}_2[0,T]}$

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Worst-case impact of stealthy attacks

- Given a protected performance ρ, the defender selects agent d and the adversary selects agent a

$$\gamma_\rho^\star(a,d) \triangleq \sup_{\zeta \in \mathcal{L}_{2e},\ \text{zero init. states}} \quad \|y_\rho\|^2_{\mathcal{L}_2[0,T]} \qquad \text{s.t.} \quad \|\eta_d\|^2_{\mathcal{L}_2[0,T]} \le \delta^2 \tag{1}$$

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- If (1) is feasible, obtain finite $\gamma_\rho^\star(a,d)$ by solving

$$\gamma_\rho^\star(a,d) \triangleq \min_{\gamma_\rho \in \mathbb{R}_+, F=F^\top \geq 0} \gamma_\rho \quad \text{s.t.} \quad R\big(\Sigma_{\text{closed-loop}}, F, \gamma_\rho\big) \leq 0,$$

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Problem: The defender selects d such that $\gamma_\rho^\star(a,d) < \infty$

Outline

Worst-case impact analysis

- Invariant zeros of system $\bar{\Sigma} \triangleq (\bar{A}, \bar{B}, \bar{C}, 0)$

$$\begin{bmatrix} \lambda I - \bar{A} & -\bar{B} \\ \bar{C} & 0 \end{bmatrix} \begin{bmatrix} \bar{x} \\ g \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \bar{x} \neq 0. \qquad (2)$$

$\lambda < \infty$: finite invariant zero
$\lambda = 1/s,\ s = 0$: infinite invariant zero
Input of $\bar{\Sigma}:\ ge^{\lambda t}$, output of $\bar{\Sigma} \to 0$

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$$\gamma_\rho^\star(a,d) \triangleq \sup_{\zeta \in \mathcal{L}_{2e},\ \text{zero init. states}} \|y_\rho\|_{\mathcal{L}_2}^2 \quad \text{s.t.} \quad \|\eta_d\|_{\mathcal{L}_2}^2 \leq \delta^2$$

- λ_d of Σ_d (Re$[\lambda_d] > 0$) is also invariant zero of Σ_ρ if, and only if, $\gamma_\rho^\star(a,d) < \infty$

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- **Finite invariant zeros λ_d of Σ_d** $<\infty$ (Re$[\lambda_d] > 0$)

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Finite unstable invariant zeros λ_d of Σ_d can be excluded by proper local control parameters. Then, $\gamma_\rho^\star(a,d) < \infty$.

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- **Infinite invariant zeros λ_d** $= 1/s$ **where** $s = 0$

Theorem 3.1 (relative degree condition)

If $r_{(d,a)} \leq r_{(\rho,a)}$, then, λ_d is also infinite invariant zero of Σ_ρ, leading to $\gamma_\rho^\star(a,d) < \infty$.

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- Admissible detection agents for the defender fulfill

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$$\max_{\mathfrak{q}(a)} \min_{\mathfrak{p}(d)} \sum_{a\in\mathcal{V}_{-\rho}} \sum_{d\in\mathcal{D}} \mathfrak{p}(d)\gamma_\rho^\star(a,d)\mathfrak{q}(a)$$

$$\text{s.t.} \sum_{a\in\mathcal{V}_{-\rho}} \mathfrak{q}(a) = 1, \quad \sum_{d\in\mathcal{D}} \mathfrak{p}(d) = 1, \text{ (mixed-strategy equilibrium)}$$

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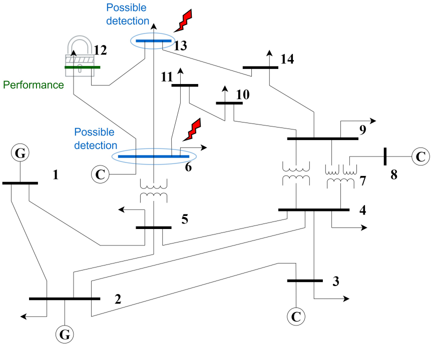
Simulation results



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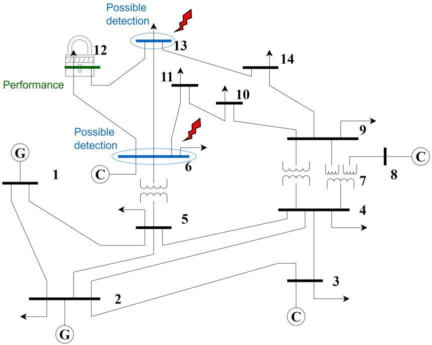

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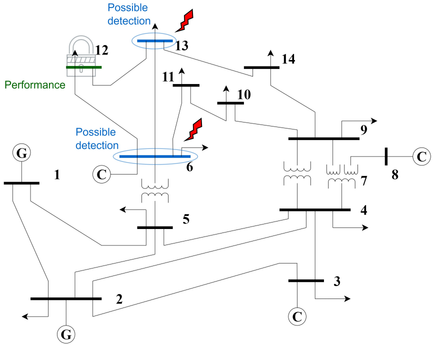

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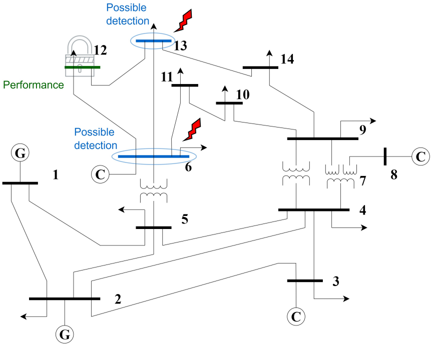

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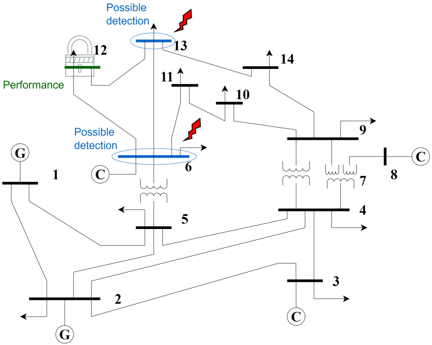

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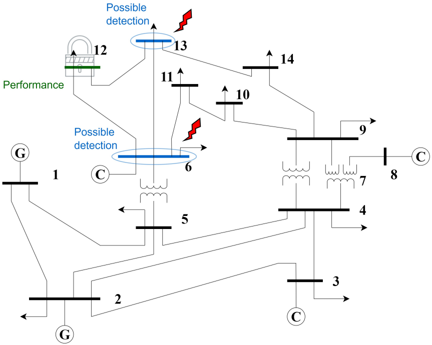

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- How can the adversary launch stealthy attacks on the network?

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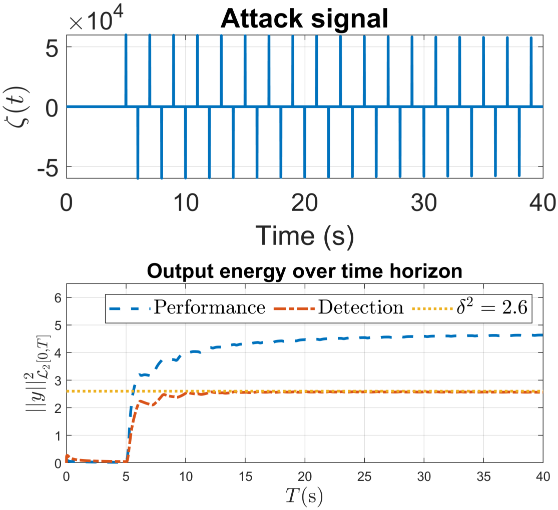
Numerical results (Cont.)



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Conclusion and future work

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- The worst-case impact of stealthy attacks is intensively investigated
- Control design and sufficient (relative degree) condition are proposed
- Admissible strategies for the defender are characterized
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Future work

- Keep the performance agent secret
- Re-design detector parameters to minimize the risk
- . . .

Thanks for your listening!!!
Questions?