



UPPSALA  
UNIVERSITET



Uppsala  
Secure Learning  
and Control Lab

# Security Allocation in Networked Control Systems

**Anh Tung Nguyen**

Dissertation for the degree of Licentiate

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STIFTELSEN för  
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Research Council

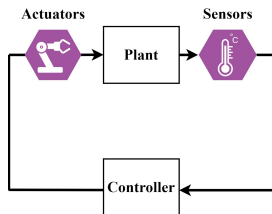
# Outline

- 1 Introduction
- 2 Security in Networked Control Systems
- 3 Problem Formulation
- 4 Contributions
  - Paper I
  - Paper II
  - Paper III
  - Paper IV
- 5 Conclusion and Future Work

# Critical Infrastructure

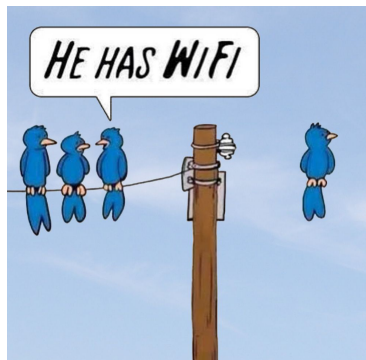
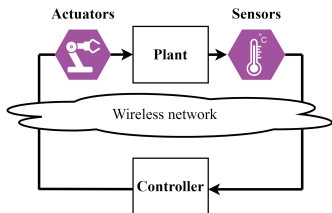
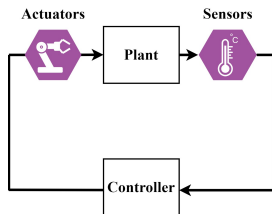


# Control of Critical Infrastructure

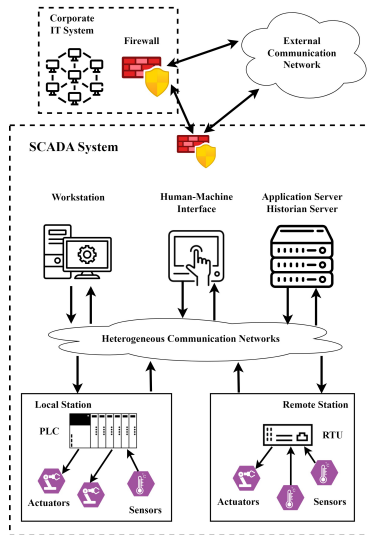
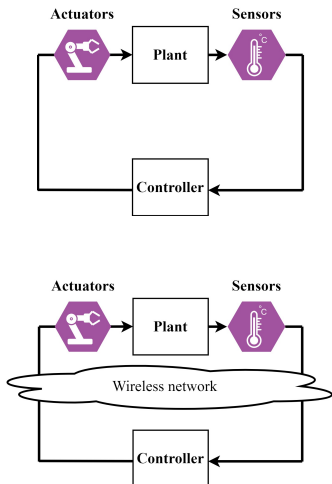




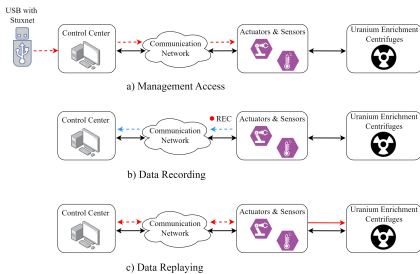
# Control of Critical Infrastructure



# Control of Critical Infrastructure



# Vulnerabilities in Critical Infrastructure



## Stuxnet





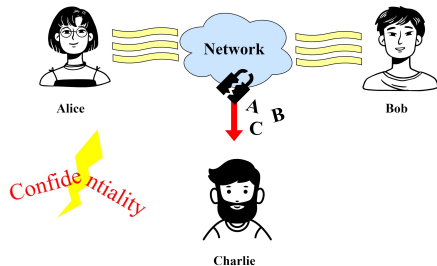
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# Security Triad and Threats

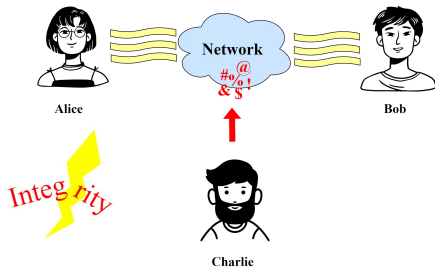
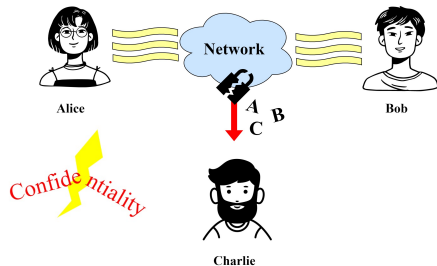


# Security Triad and Threats

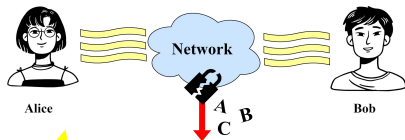




# Security Triad and Threats



# Security Triad and Threats



Confidentiality



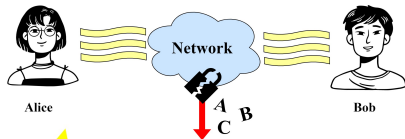
Integrity



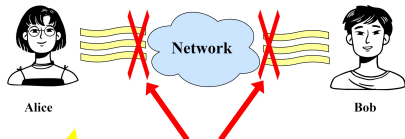
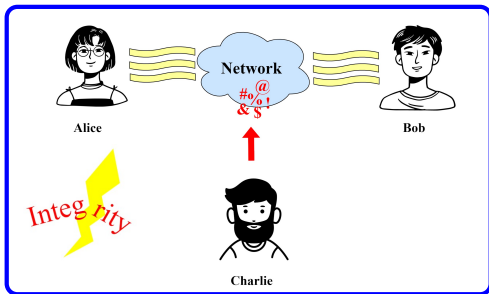
Availability



# Security Triad and Threats



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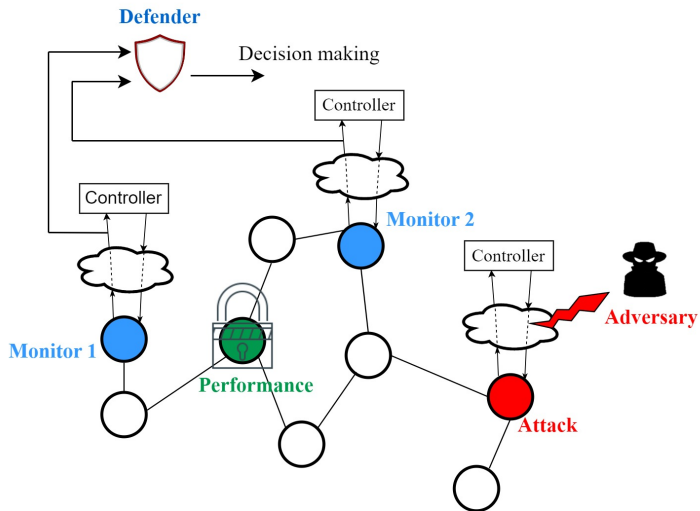
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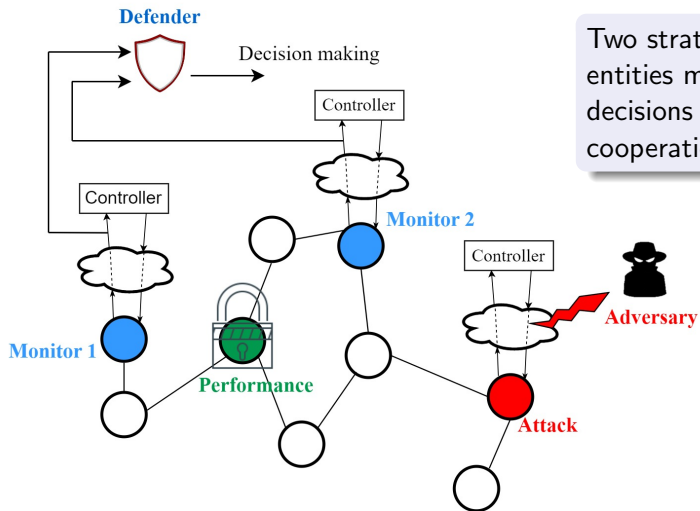
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# Problem description



# Problem description



Two strategic entities make decisions without cooperation

# Problem analysis



Defender

- Purpose: protect the system



Adversary

- Purpose: attack the system

# Problem analysis



Defender

- Purpose: protect the system
- Action: monitor what?



Adversary

- Purpose: attack the system
- Action: attack what?



# Problem analysis



Defender

- Purpose: protect the system
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Adversary

- Purpose: attack the system
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Action order:

- 1) Make decisions simultaneously
- 2) The defender goes first

# Problem analysis



Defender

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Which move to be the GOAT



Non-cooperative  
two-player game

# Problem analysis



Defender

- Purpose: protect the system
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Action order:

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Adversary

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## Problem variations

- ▷ System models
- ▷ Resources & knowledge
- ▷ Action order

# Problem variations



Defender



Adversary

Performance  $\rho$ 

## Paper I

- **Certain** LFO
- Performance  $\rho$  is **fixed**
- Def./Adv. chooses **one**
- Take actions **simultaneously**

## Paper II

- **Uncertain** LFO
- Performance  $\rho$  is **fixed**
- Def./Adv. chooses **one**
- Take actions **simultaneously**

## Paper III

- **Certain** LSO
- Performance  $\rho$  is **fixed**
- Def./Adv. chooses **one**
- Take actions **simultaneously**

## Paper IV

- **Certain** LFO
- Performance  $\rho$  is **uncertain**
- Adv. chooses **one**, Def. chooses **several**
- Def. takes action **firstly**

# Problem variations



Defender



Adversary

Performance  $\rho$ 

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# Problem formulation

- Undirected connected graph  $\mathcal{G}$  with  $N$  nodes

$$\dot{x}_i(t) = A_i x_i(t) + b \tilde{u}_i(t),$$

$$y_i(t) = c^\top x_i(t).$$



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- Undirected connected graph  $\mathcal{G}$  with  $N$  nodes

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- Local performance:  $\|y_\rho\|_{\mathcal{L}_2[0,T]}^2 = \frac{1}{T} \int_0^T |y_\rho(t)|^2 dt$

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Adversary  
chooses node  $a$

- Local performance:  $\|y_\rho\|_{\mathcal{L}_2[0,T]}^2 = \frac{1}{T} \int_0^T |y_\rho(t)|^2 dt$
- Healthy/attacked local controller

$$\tilde{u}_i(t) = \underbrace{\sum_{j \in \mathcal{N}_i} \phi_{ij}(x_i, x_j)}_{\text{healthy}} + \begin{cases} 0, & \text{if } i \neq a \\ \zeta(t), & \text{if } i \equiv a \end{cases}$$

$\Rightarrow$  Closed-loop system:  $\dot{x}(t) = Ax(t) + b \otimes e_a \zeta(t)$

## Problem formulation (Cont.)

- Closed-loop system:

$$\dot{x}(t) = Ax(t) + b \otimes e_a \zeta(t), \quad x(0) = 0$$

## Problem formulation (Cont.)

- Closed-loop system:

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- The defender can choose several nodes  $\mathcal{M} = \{m_1, m_2, \dots, m_{|\mathcal{M}|}\}$

$$y_{m_1}(t) = e_{m_1}^\top x(t), \quad y_{m_2}(t) = e_{m_2}^\top x(t), \quad \dots \quad y_{|\mathcal{M}|}(t) = e_{|\mathcal{M}|}^\top x(t).$$

- Monitor outputs such that at least

$$\|y_{m_k}\|_{\mathcal{L}_2}^2 = \frac{1}{T} \int_0^T |y_{m_k}(t)|^2 dt > \delta_{m_k} \quad \Rightarrow \quad \text{Attack is detected!!!}$$

## Problem formulation (Cont.)

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- Adversary's purpose: **stay stealthy**

$$\|y_{m_k}\|_{\mathcal{L}_2}^2 = \frac{1}{T} \int_0^T |y_{m_k}(t)|^2 dt \leq \delta_{m_k} \quad \forall m_k \in \mathcal{M}$$

$\Rightarrow$  **Stealthy False Data Injection Attacks (Stealthy FDI Attacks)**

## Challenges

Attack      Monitor

Performance

$$J_\rho(a, \mathcal{M}) \triangleq \sup_{\zeta \in \mathcal{L}_{2e}, \text{ zero init. state}} \|y_\rho\|_{\mathcal{L}_2}^2 \quad (1)$$

s.t.  $\|y_{m_k}\|_{\mathcal{L}_2}^2 \leq \delta_{m_k}, \forall m_k \in \mathcal{M}$

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Combinatorial optimization problem

## Challenges

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Combinatorial optimization problem

$\Rightarrow$

Computational burden

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Shrink defender action space  $\mathcal{M} \subset \mathbb{D} \subset \mathcal{V}$

## Challenges

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Combinatorial optimization problem

$\Rightarrow$

Computational burden

Shrink defender action space  $\mathcal{M} \subset \mathbb{D} \subset \mathcal{V}$

$\Rightarrow$

Efficiently allocate  
defense resources

# Challenges

$$\begin{array}{c}
 \text{Attack} \quad \text{Monitor} \\
 \swarrow \quad \searrow \\
 J_\rho(a, \mathcal{M}) \triangleq \\
 \uparrow \\
 \text{Performance}
 \end{array}
 \sup_{\zeta \in \mathcal{L}_{2e}, \text{ zero init. state}} \|y_\rho\|_{\mathcal{L}_2}^2 \tag{1}$$

s.t.  $\|y_{m_k}\|_{\mathcal{L}_2}^2 \leq \delta_{m_k}, \forall m_k \in \mathcal{M}$

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Combinatorial optimization problem



Computational burden

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Efficiently allocate defense resources



$\mathbb{D}$  guarantees the boundedness of (1)

# Challenges

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Combinatorial optimization problem

⇒ Computational burden

Shrink defender action space  $\mathcal{M} \subset \mathbb{D} \subset \mathcal{V}$

⇒ Efficiently allocate defense resources



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← **Main contributions**  
Characterize  $\mathbb{D}$

## Preliminary results

Boundedness of the worst-case impact of stealthy FDI attacks



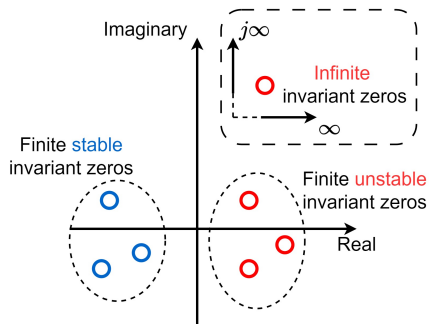
Invariant zeros

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Invariant zeros





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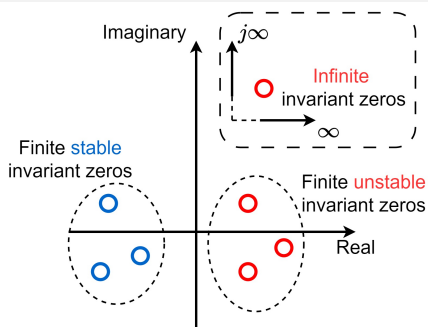
⇔ Invariant zeros

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- Systems  $\Sigma_\rho = (A, b \otimes e_a, e_\rho^\top, 0)$  and  $\Sigma_{m_k} = (A, b \otimes e_a, e_{m_k}^\top, 0)$

$$J_\rho(a, \mathcal{M}) \triangleq \sup_{\zeta \in \mathcal{L}_{2e}, \text{ zero init. state}} \|y_\rho\|_{\mathcal{L}_2}^2$$

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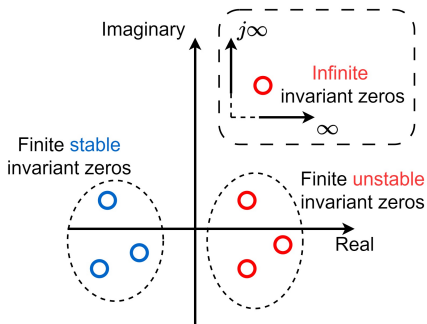


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$$\text{s.t.} \quad \|y_{m_k}\|_{\mathcal{L}_2}^2 \leq \delta_{m_k}, \quad \forall m_k \in \mathcal{M}$$

- At least  $\Sigma_{m_k}$ , its  $\lambda_{m_k}$  ( $\text{Re}[\lambda_{m_k}] > 0$ ) is also invariant zero of  $\Sigma_\rho$ ,

if, and only if,  $J_\rho(a, \mathcal{M}) < \infty$

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# Paper I - Problem variation



Defender



Adversary

Performance  $\rho$ 

## Paper I

- Certain LFO
- Performance  $\rho$  is fixed
- Def./Adv. chooses one
- Take actions simultaneously

## Paper II

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# Challenges

- **Unweighted** graph  $\mathcal{G}$  with  $N$  vertices, **certain** Laplacian matrix  $L$

$$\dot{x}(t) = -Lx(t) + e_a \zeta(t),$$

$$y_\rho(t) = e_\rho^\top x(t),$$

$$y_m(t) = e_m^\top x(t) \quad (\mathcal{M} = \{m\}).$$

- Worst-case impact of stealthy FDI attacks

$$J_\rho(a, m) \triangleq \sup_{\zeta \in \mathcal{L}_{2e}, \text{ zero init. state}} \|y_\rho\|_{\mathcal{L}_2}^2$$

$$\text{s.t.} \quad \|y_m\|_{\mathcal{L}_2}^2 \leq \delta_m$$

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No finite unstable  
invariant zeros<sup>1</sup>

1. J. A. Torres & S. Roy, "Graph-theoretic analysis of network input-output processes: Zero structure and its implications on remote feedback control", *Automatica*, 2015

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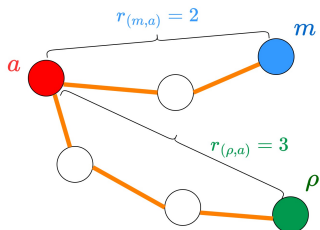
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Challenge

Infinite invariant zeros

1. J. A. Torres & S. Roy, "Graph-theoretic analysis of network input-output processes: Zero structure and its implications on remote feedback control", *Automatica*, 2015

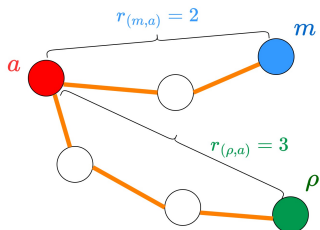




## Main results

$\Sigma_m$ : output at  $m$ , relative degree  $r(m,a)$

$\Sigma_\rho$ : output at  $\rho$ , relative degree  $r(\rho,a)$

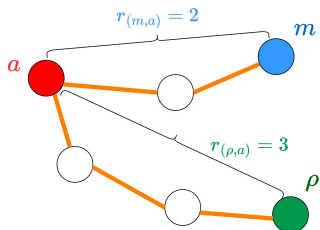


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# inf. inv. zero = relative degree



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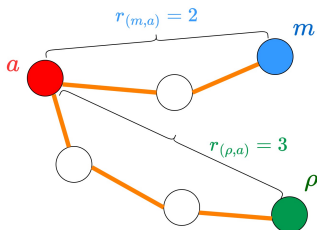
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### Theorem 1

# inf. inv. zero of  $\Sigma_m \leq$  # inf. inv. zero of  $\Sigma_\rho$

$$\Leftrightarrow r(m,a) \leq r(\rho,a) \Leftrightarrow J_\rho(a, m) < \infty$$



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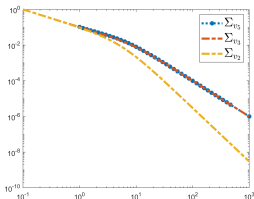
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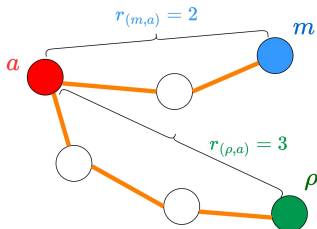
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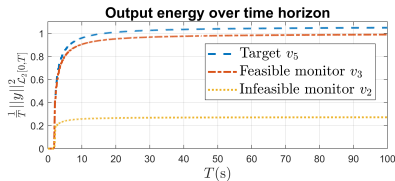
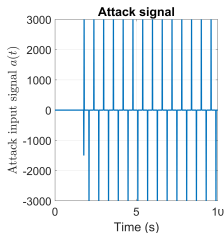
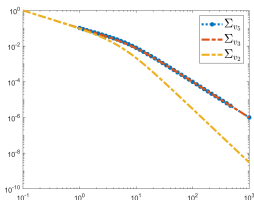
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# Outline

- 1 Introduction
- 2 Security in Networked Control Systems
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  - Paper I
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# Paper II - Problem variation



Defender



Adversary

Performance  $\rho$ 

## Paper I

- **Certain** LFO
- Performance  $\rho$  is fixed
- Def./Adv. chooses one
- Take actions simultaneously

## Paper II

- **Uncertain** LFO
- Performance  $\rho$  is **fixed**
- Def./Adv. chooses **one**
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## Paper III

- **Certain** LSO
- Performance  $\rho$  is fixed
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## Paper IV

- **Certain** LFO
- Performance  $\rho$  is uncertain
- Adv. chooses one, Def. chooses several
- Def. takes action first

# Challenges

- **Uncertain weighted** graph  $\mathcal{G}$  with  $N$  vertices, **uncertain**  $L^\Delta$

$$\dot{x}^\Delta(t) = -L^\Delta x^\Delta(t) + e_a \zeta(t),$$

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$$L^\Delta = \bar{L} + \Delta$$

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- 1) Finite unstable inv. zeros<sup>1</sup>
- 2) Infinite inv. zeros
- 3) Evaluate worst-case attack impact

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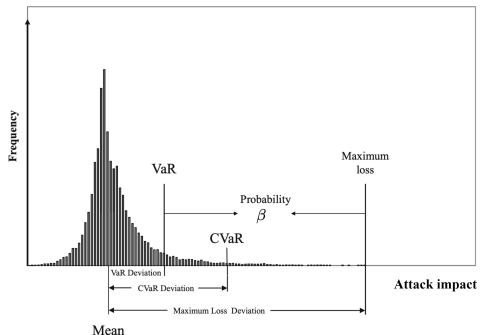
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# Value-at-Risk

$$\mathcal{J}_\rho(a, m) = \text{VaR}_{\beta, \Omega} \left[ \sup_{\zeta \in \mathcal{L}_{2e}} J_\rho(a, m; \Delta, \zeta) \right]$$

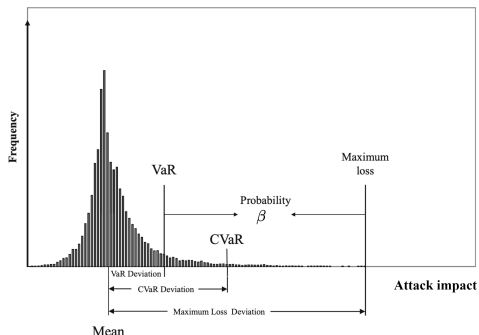
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## Theorem 1 & Lemma 2

$M_1$  values from  $\Omega$

Evaluate  $\lceil M_1(1 - \beta_1) \rceil$  values  
with  $\epsilon$  accuracy

$$M_1 \geq \frac{1}{2\epsilon_1^2} \log \frac{2}{\beta_1}$$

E.g.,  $\epsilon_1 = 0.06$ ,  $\beta_1 = 0.08$ ,

$$M_1 \geq 450 \Rightarrow 414 \text{ values}$$

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# Challenges

- **Main focus:** Power networks by linearized swing equations

$$m_i \ddot{p}_i(t) + h_i \dot{p}_i(t) = \sum_{j \in \mathcal{N}_i} \ell_{ij} (p_i(t) - p_j(t)) + \tilde{u}_i(t),$$

- Closed-loop system

$$\dot{x}(t) = Ax(t) + e_a \zeta(t),$$

$$y_i(t) = C_i x(t), \quad \forall i \in \mathcal{V},$$

$$y_\rho(t) = C_\rho x(t),$$

- Local performance:  $\|y_\rho\|_{\mathcal{L}_2[0,T]}^2 = \frac{1}{T} \int_0^T |y_\rho(t)|^2 dt$

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Detector

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# Main results

- The worst-case impact of stealthy FDI attacks

$$J_\rho(a, m) \triangleq \sup_{\zeta \in \mathcal{L}_{2e}, \text{ zero init. states}} \|y_\rho\|_{\mathcal{L}_2}^2$$

s.t.  $\|\eta_m\|_{\mathcal{L}_2}^2 \leq \delta$

- Systems  $\Sigma_\rho = (A, e_a, C_\rho, 0)$  and  $\Sigma_m = (A, e_a, C_m, 0)$
- Denote  $r_{(\rho, a)}$  and  $r_{(m, a)}$  as the relative degrees of  $\Sigma_\rho$  and  $\Sigma_m$

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## Theorem 3.1 (relative degree condition)

$$r_{(m, a)} \leq r_{(\rho, a)}$$

$$\Rightarrow J_\rho(a, m) < \infty$$

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# Paper IV - Problem variation



Defender



Adversary

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- Worst-case attack impact

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# Players' strategies

## Defender strategy

$$\mathcal{M}^* = \arg \min_{\mathcal{M} \subseteq \mathbb{D}} \text{Defense cost} |_{a^*(\mathcal{M})}$$

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Combinatorial optimization problem

Defender

Adversary



Computational burden

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Combinatorial optimization problem



Computational burden

Shrink defender action space  $\mathcal{M} \subset \mathbb{D} \subset \mathcal{V}$ Efficiently allocate  
defense resources $\mathbb{D}$  s.t. defense cost/attack impact  $< \infty$

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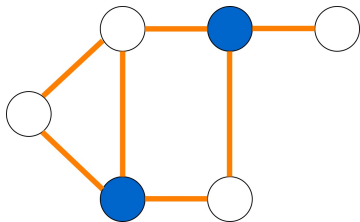
Combinatorial optimization problem



Computational burden

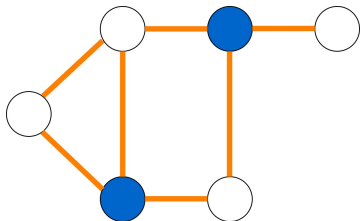
Shrink defender action space  $\mathcal{M} \subset \mathbb{D} \subset \mathcal{V}$ Efficiently allocate  
defense resources $\mathbb{D}$  s.t. defense cost/attack impact  $< \infty$ Characterize  $\mathbb{D}$

# Main results



Dominating set

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Dominating set

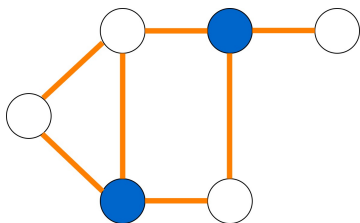
## Theorem 2 (necessary and sufficient condition)

$\mathcal{M}$  is a dominating set

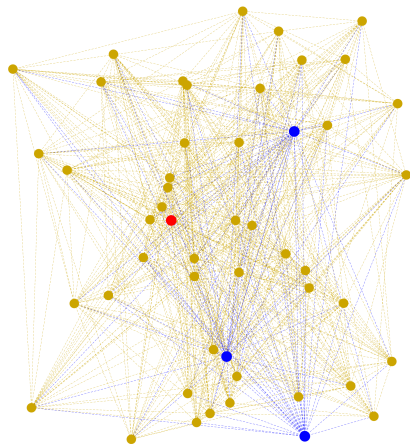
$\Leftrightarrow$  def. cost  $R(a, \mathcal{M}) < \infty$

& attack impact  $Q(a, \mathcal{M}) < \infty$

# Main results



Dominating set



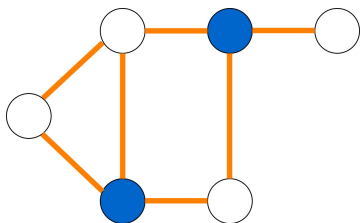
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# Main results



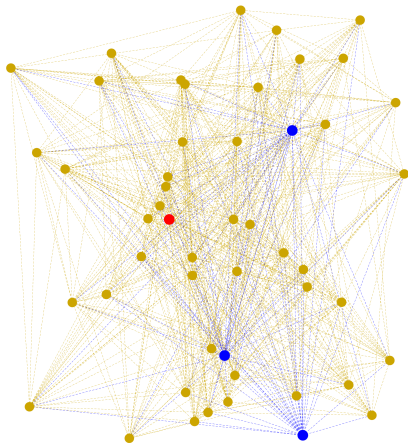
Dominating set

## Theorem 2 (necessary and sufficient condition)

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$\Leftrightarrow$  def. cost  $R(a, \mathcal{M}) < \infty$

& attack impact  $Q(a, \mathcal{M}) < \infty$



$$R(a, \mathcal{M}) \leq 50.2456$$

$$Q(a, \mathcal{M}) \leq 48.4235$$



# Outline

- 1 Introduction
- 2 Security in Networked Control Systems
- 3 Problem Formulation
- 4 Contributions
  - Paper I
  - Paper II
  - Paper III
  - Paper IV
- 5 Conclusion and Future Work

# Conclusion and Future Work

## This Licentiate thesis has

- ① considered several types of NCSs under attacks
- ② intensively investigated the worst-case impact of stealthy FDI attacks
- ③ found system- and graph-theoretic conditions
- ④ assisted the defender in allocating defense resources

# Conclusion and Future Work

## This Licentiate thesis has

- 1 considered several types of NCSs under attacks
- 2 intensively investigated the worst-case impact of stealthy FDI attacks
- 3 found system- and graph-theoretic conditions
- 4 assisted the defender in allocating defense resources

## Toward the PhD thesis, it will be extended to

- 1 overcome combinatorial optimization problem
- 2 consider uncompleted information
- 3 consider multiple adversaries
- 4 assist the defender in designing detectors
- 5 . . . . .



Defender



Adversary

Performance  $\rho$ 

### Paper I

- **Certain** LFO
- Performance  $\rho$  is **fixed**
- Def./Adv. chooses **one**
- Take actions **simultaneously**

### Paper II

- **Uncertain** LFO
- Performance  $\rho$  is **fixed**
- Def./Adv. chooses **one**
- Take actions **simultaneously**

### Paper III

- **Certain** LSO
- Performance  $\rho$  is **fixed**
- Def./Adv. chooses **one**
- Take actions **simultaneously**

### Paper IV

- **Certain** LFO
- Performance  $\rho$  is **uncertain**
- Adv. chooses **one**, Def. chooses **several**
- Def. takes action **first**

Thanks for listening!!!